Final Project

Monte Carlo Markov Chain Simulation
To Calculate Elevator's Round Trip Time under incoming
traffic conditions

UNIVERSITY OF JORDAN
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Mechatronics Engineering Department
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Supervisor
Dr. Lutfi Rawhi Al-sharif

By
Hasan Shaban Algzawi

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Certificate

Certifies that the work contained in this report entitled:

Was carried out by

Hasan Shaban Algzawi

Under my supervision and that in my opinion, it is fully adequate, in scope and quality, for the requirements of the graduation project in Mechatronics Engineering Department.

Supervisor

Dr. Lutfi Rawhi Al-sharif

Mechatronics Engineering Department

Faculty of Engineering and Technology

University of Jordan, Amman, Jordan

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RTT: Round Trip Time.

MCMC: Monte Carlo Markov Chain.

MC: Monte Carlo.

PDF: Probability Density Function.

CDF: cumulative Distribution function.

HC: Handling Capacity.

Int.: Interval.

CC: Car Capacity.

GUI: Graphical User Interface.

s: Seconds.

$\boldsymbol{\tau}$**:** round trip time.

L: number of elevators.

N: the total number of floors.

U: population of the building.

U_i**:** The ith floor population.

P: number of passengers inside the elevator.

S: Probable number of stops.

H: Highest reversal floor.

***Pr*:** Probability.

P_{ij}**:** The probability of going from state i to state j in n time steps.

J: journey.

$P(J_{ij})$: The probability of the elevator's making a journey between floor *i* and floor j

$P(S_i)$: The probability of the elevator's stopping at floor i

d: distance traveled [m]

v: The rated speed [m/s]

a: Acceleration [m/s^2]

j: Jerk [m/s^3]

τ_k: Kinematics time [s].

t_{ij}: Time needed for the elevator's transition between two floors [s]

d_f: finished floor to finished floor level [m]

t_{do}: Door opening time. [s]

t_{dc}: Door closing time. [s]

t_{pi}: Passenger's boarding time. [s]

t_{po}: Passenger's alighting time. [s]

τ_i: the round trip time found in the ith trial [s]

τ_G: The time spent at the ground floor [s]

τ_S: The time spent travelling to the upper destination floors and delivering the passengers [s]

τ_H: The time spent returning back to the ground terminal from the highest reversal floor [s]

t_{sd}: Motor start delay [s]

t_{ao}: Advance door opening time [s]

t_{acc}: the time taken to accelerate up to the top speed from standstill [s]

t_{dec}: the time taken to decelerate down from the top speed down to standstill [s]

$e\%$: percentage of error.

Abstract

Nowadays the challenge of the development of elevator systems involves the improvement of the quality and quantity of service to provide the passengers the minimum waiting or travelling time, or to reduce the consumption of power in a group of elevators system through elevator traffic analysis. The design of elevator system is based on determining the number, speed and capacity of elevators. Elevator traffic analysis depends mainly on the calculation of the Round trip time, therefore in this report a Monte Carlo Markov Chain simulation method is introduced to calculate the round trip time under incoming (up-peak) traffic conditions and with the assumptions of equal number of floor population, equal floor heights and a top speed that is attained in a one floor journey.

Monte Carlo Markov Chain simulation method is a numerical probabilistic method based on a large number of trials to approach the exact value. The availability of powerful computing programs that are easily accessed by computers and laptops, that is spread everywhere and exist almost in every house, made it practical and easy to use the Monte Carlo Markov Chains simulation in evaluating the round trip time value of elevator systems, where it is easy to run tens of thousands of simulation runs within fractions of a second.

Another simulation method; Monte Carlo Simulation method, is also presented in this report, where the results of this method is compared to the Monte Carlo Markov chains simulation method. Results of the simulation show that the Monte Carlo Markov Chains method is better than the Monte Carlo method where the Monte Carlo Markov chains method gives very accurate value and with easiness of software simulation and a less number of trials compared to the number of trials required for satisfactory result using the Monte Carlo Simulation method.

The power of the Monte Carlo Markov Chains simulation method is that it is easy to develop it for calculating the round trip time if either one or more of the assumptions of equal floor heights, equal floor population and that the top speed is attained in a one floor journey were dropped.

Chapter One

Introduction to the design of elevator systems

1.1　History of elevators

The elevator is a lifting device that is moved vertically to transport people or things up or down along a vertical shaft. The shaft is usually made of cables, motor and the operating equipments. See figure 1.1.

Figure 1.1: Elevator's shaft

Elevators were developed through history from a simple lift powered by animals, water wheels power or even by hand to a lift powered by hoist and afterwards powered by a steam. Then the development of elevators continued to after that becomes powered by electricity. The development of industries and the need for transporting of materials in factories was mainly the reason of developing of elevators systems.[1]. Also, the production of Electrical elevators revolutionized the use of elevators in industries and buildings up to this level that we have today, see figure 1.2.

Figure 1.2: An old and a new kind of elevators

1.2 Elevator Traffic Analysis

1.2.1 Elevator traffic analysis definition

Elevator traffic analysis is studying and analyzing the performance of a group of elevators based on assumptions about the expected traffic situation.

The objective of elevator traffic analysis is to find the suitable elevators that satisfy the performance desired for a building; number, velocity and capacitance of elevators will be determined.

The main performance measurements are quantity of service and quality of service, where the quantity of service refers to the handling capacity and the quality of service refers to the interval.

- Handling Capacity is the percentage of the building population that the group of elevators can support in a given time period (usually in 5 minutes).

- Interval is the average time between the arriving of tow elevators, and it can be a good measure of the waiting time of the passengers at the main floor. see figure.1.3.

Figure 1.3: Waiting time

The elevator traffic analysis is based on assumptions about the movement of the building population, for example when and where do they enter or leave and are there facilities like restaurants, gyms…etc in the building that affects the usual passengers flowing in and out.

If the assumptions of population movement in the building were accurate, then the results of the traffic analysis will be accurate too, because then we can make an accurate calculation of the elevator performance.

The outputs of a traffic analysis will give an accurate indication of the quality and quantity of the elevator service provided, like the calculation of waiting times and the percentage of the building population that can be transported by the elevators in a given period of time.

Elevator traffic analysis is used in the design of new buildings or existing buildings. In new buildings it is used in the design of elevators to know the size, speed and capacity of elevators needed to provide the quantity and quality levels of service required.

For existing buildings elevator traffic analysis are used to predict the effect of changes in a building's population or configuration on the elevator service, because the population of building is often increased and sometimes some features are added to it or changed like restaurants, gyms…etc. These changes can affect the flow of people [2].

There are four main types of elevator traffic modes;

1- Up peak traffic.

2- Down peak traffic.

3- Lunch time (two way) traffic.

4- Inter-floor traffic.

In this research, we will focus on the incoming (up-peak) traffic, which states that the passengers arrives at the main terminal and then being transported to the upper floors, then the elevator returns to the terminal floor to pick up passengers again to the upper floors.

1.2.2   Round trip time

1.2.2.1   Definition of Round trip time

The round trip time (RTT) is the time needed for a single elevator to complete a closed path in a building, or the time needed for the passengers to travel from the main floor (ground floor) to the highest reversal floor and back to the main floor.

The round trip time starts from the door opening at main floor and ends with door opening, see figure 1.4. The highest reversal floor is the highest floor that the elevator reaches in one journey.

The round trip time is the most important parameter in the design of modern elevator systems and in elevator traffic analysis. Modern buildings are becoming more complicated and more sophisticated. Therefore, it is an important task to develop accurate methods for the calculation of the round trip time that catches up with the development of the modern buildings.
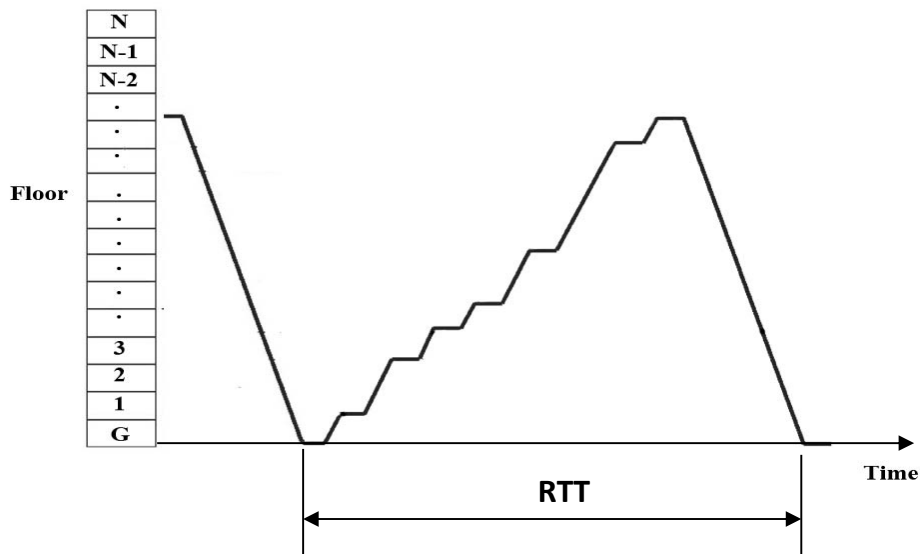
Figure 1.4: the round trip time timeline during up-peak traffic

It is clear from the graph that the round trip time in the up peak traffic contains four components; the time needed to collect passengers at the main floor, the time needed to deliver passengers to their destinations, the time spent in stopping at each destination floor and the time spent by the elevator while returning to the main floor.

1.2.2.1 Importance and uses of Round trip time

The RTT is important because it is used to indicate the elevator's performance, because it is related to the handling capacity as shown in the following equation.

$$HC\% = \frac{300 \cdot L \cdot P}{\tau \cdot U}$$

(1.1)[10]

Where :

HC%: handling capacity.

L : number of elevators.

P: number of passengers inside the elevator.

τ: round trip time.

U: population of the building.

Also the round trip time is used to measure the interval [3], because the interval is calculated by dividing the round trip time by the number of elevators , see the following equation.

$$int = \frac{\tau}{L}$$

(1.2)[10]

Where:

L : number of elevators.

τ: round trip time.

Usually the performance of elevator system is determined in the incoming traffic situation, where passengers move from the ground floor to upper floors. That is because the incoming traffic is a difficult traffic situation. There are many methods for calculating the RTT in the incoming traffic situation. The simplest method is based on calculating the expected number of stops and the expected highest reversal floor and substituting them in the RTT equation [3].

For more accurate result we should not just calculate the number of stops, but also we should calculate the probability of flow between each pair of floors for incoming traffic. Hence we will introduce another two methods for calculating round trip time which are the Markov Chain method and Monte Carlo Markov Chain method. Before discussing these three methods mentioned above we will give a brief description of the meaning of Markov chains and Monte Carlo Markov Chains.

Chapter two

Introduction to Monte Carlo Markov Chain

2.1 Markov Chains

A Markov chain, is a mathematical system that undergoes transitions from one state to another, see figure 2.1. The number of possible states is finite or countable. Markov chain is a random process usually described as memoryless; meaning that the next state depends only on the current state and does not depend on the sequence of events after it. This "memorylessness" is called the Markov property. Therefore, a Markov chain can be described as a random process with the Markov property. Markov chains are very useful and have many applications as statistical models of real-world processes.
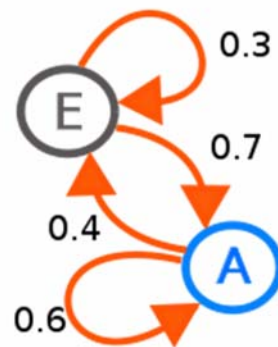
Figure 2.1: Markov Chain

Often, the term "Markov chain" is used to mean a Markov process which has a discrete state-space, meaning that the number of states is finite or countable. Usually a Markov chain is defined for a discrete set of times (it is expressed as " a discrete-time Markov chain")[4] although some authors use the same expression where "time" can take continuous values.[5,6] The use of the term in Markov chain Monte Carlo method covers cases where the process is in discrete time (discrete algorithm steps) with a continuous state space.

A discrete-time random process is a system which is in a certain state at each step, by going from one discrete step to another the state changes. The steps are often moments in time, but they can also refer to physical distance or any other discrete measurement; in other words, the steps are the integers, and the random process is a mapping of these steps to states. The Markov property means that the probability for the system at the next step depends only on the current state of the system, and not on the state of the system at previous steps.

The changes of states of the system are called 'transitions', and the probabilities of the various state-changes are called 'transition probabilities'. The set of all states and transition probabilities completely makes a Markov chain. By convention, we assume all possible states and transitions have been included in the definition of the processes, so there is always a next state and the process goes on forever.

The system changes randomly, therefore it is impossible to predict with certainty the state of a Markov chain at a given point in the future. But the statistical properties of the system's future can be predicted. It is these statistical properties that are important.

2.1.1 Markov Chain Formal definition

A Markov chain is a sequence of random variables X_1, X_2, X_3, ... with the Markov property, such that, given the present state, the future and past states are independent. Formally,

$$\Pr(X_{n+1} = x | X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x | X_n = x_n). \quad (2.1)$$

Where:

Pr: Probability.

The possible values of X_i form a countable set S called the state space of the chain. Markov chains are often described by a directed graph, where the edges are labeled by the probabilities of going from one state to the other states as seen before in figure 2.1. The probability of going from state i to state j in n time steps is

$$p_{ij}^{(n)} = \Pr(X_n = j \mid X_0 = i) \qquad (2.2)$$

Note: The superscript (n) is an index and not an exponent.

Where:

P_{ij} : is the probability of going from state i to state j in n time steps.

The single-step transition is

$$p_{ij} = \Pr(X_1 = j \mid X_0 = i). \qquad (2.3)$$

2.1.2 Example of a Markov chain

Markov chains are used in music composition or making a song melody. The states of the system are the note values and there's a probability vector for each note, see figure2.2. Completing a transition probability matrix, see Table 2.1. An algorithm is constructed to produce an output note values based on the transition matrix probabilities. [7]
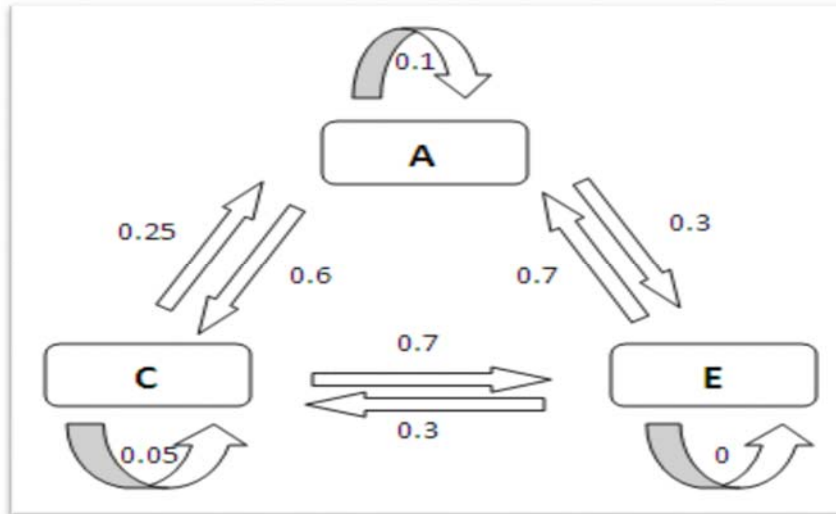
Figure 2.2: Markov chain for music notes

Table2.1: transition probability matrix for the music notes A, C, E

Note	A	C	E
A	0.1	0.6	0.3
C	0.25	0.05	0.7
E	0.7	0.3	0

2.1.3 Applications of Markov Chains

Markov chains are applied to many different fields, some of them are:

- Physics
- Medicine
- Information sciences
- Queueing theory
- Internet applications
- Social sciences
- Chemistry
- Games
- Music
- Statistics

2.2   Monte Carlo Markov Chains

Markov chain Monte Carlo (MCMC) methods are a class of algorithms for sampling using probability distributions. It is based on constructing a Markov chain that has the desired distribution as its stationary distribution. The state of the chain after a large number of steps is then used as a sample of the desired distribution. As the number of steps increases, the quality of the sample improves.

It is not hard to construct a Markov chain with the desired properties. The difficult thing is to determine how many steps are needed to get close to the stationary distribution with a small error. A good chain will reach the stationary distribution quickly starting from any position.

2.2.1 Applications of Monte Carlo Markov Chains

MCMC methods are useful for simulating events with uncertainty in inputs and systems with a large number of coupled degrees of freedom. Fields of application include:

- Physical sciences
- Engineering
- Computational biology
- Games
- Design and visuals
- Finance and business
- Telecommunications
- Applied statistics

In this paper the application of Markov chain methods used is statistics, because we will use Markov chain Monte Carlo method to generate a sequence of random numbers to reflect the probability distribution that is desired.

Chapter three

**Monte Carlo Markov Chain and Monte Carlo simulation
methods to calculate Round trip time**

3.1 Introduction

Our objective is to calculate the round trip time under incoming traffic conditions and under the assumption of equal floor height, equal floor population, single entrance and that the top speed is attained in one floor journey. Many methods exist for calculating the round trip time; three methods are presented in this paper;

1- Monte Carlo Markov Chain simulation method.
2- Monte Carlo simulation method.
3- The analytical method.

3.2 Monte Carlo Markov Chain simulation method

Monte Carlo Markov Chain simulation method contains four main steps;

1- Forming the transition probability matrix.
2- Generation of the random journey scenario.
3- Calculation of the RTT.
4- Trials of the procedure.

3.2.1 The transition probability matrix

The Monte Carlo Markov Chain method requires the development of the transition matrix for the probability of elevator's movement between any two floors. We have two equations for the probability of elevator's transition between two floors; the probability of elevator's transition from the main floor to any upper floor, and the probability of elevator's transition between any floor except the ground floor to any other floor. The origin floor is denoted as i and the destination floor is denoted as j. meaning that the elevator moves from floor i to floor j. i and j take values from 0 (for the ground floor) to N (for the highest floor).

3.2.1.1 The probability of elevator's transition between any two floors.

To derive the equation of the probability of elevator's movement between floors i and j, the following probabilities are defined as follows.

$P\left(J_{ij}\right)$ is the probability of the elevator making a journey between floor *i* and floor j without stopping at any of the floors in between.

$P\left(S_i\right)$ is the probability of the elevator stopping at floor *i*.

$P\left(S_j\right)$ is the probability of the elevator stopping at floor *j*.

$P\left(S_{i+1,i+2,...,j-2,j-1}\right)$ is the probability of the elevator stopping at any of the floors between floors *i* and *j*.

$P\left(\overline{S_i}\right)$ is the probability of the elevator not stopping at floor *i*.

$P\left(\overline{S_j}\right)$ is the probability of the elevator not stopping at floor *j*.

$P\left(\overline{S_{i+1,i+2,...,j-2,j-1}}\right)$ is the probability of the elevator not stopping at any of the floors between floors i and j.

In order for an elevator to make a journey from floor *i* to floor *j* without stopping at any of the middle floors in between, the following statement should be true:

<div align="center">

The elevator stops at *i*

AND

The elevator stops at *j*

AND

The elevator does not stop at any of the in between floors (*i*+1, *i*+2, *i*+3....*j*-2, *j*-1)

</div>

This statement could be expressed mathematically as follows:

$$P\left(J_{ij}\right)= P(S_i)\cdot P(S_j)\cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right)$$ (3.1) [8]

This could be rewritten as:

$$P\left(J_{ij}\right)= \left(1-P\left(\overline{S_i}\right)\right)\cdot\left(1-P\left(\overline{S_j}\right)\right)\cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right)$$ (3.2) [8]

Expanding gives:

$$P(J_{ij}) = P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) \cdot \left(1 - P\left(\overline{S_i}\right) - P\left(\overline{S_j}\right) + P\left(\overline{S_i}\right) \cdot P\left(\overline{S_j}\right)\right) \qquad (3.3)\ [8]$$

$$P(J_{ij}) = P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) - P\left(\overline{S_i}\right) \cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) - P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) \cdot P\left(\overline{S_j}\right)$$
$$+ P\left(\overline{S_i}\right) \cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) \cdot P\left(\overline{S_j}\right) \qquad (3.4)\ [8]$$

But:

$$P\left(\overline{S_i}\right) \cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) = P\left(\overline{S_{i,i+1,i+2....j-2,j-1}}\right) \qquad (3.5)\ [8]$$

And:

$$P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) \cdot P\left(\overline{S_j}\right) = P\left(\overline{S_{i+1,i+2....j-2,j-1,j}}\right) \qquad (3.6)\ [8]$$

And:

$$P\left(\overline{S_i}\right) \cdot P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) \cdot P\left(\overline{S_j}\right) = P\left(\overline{S_{i,i+1,i+2....j-2,j-1,j}}\right) \qquad (3.7)\ [8]$$

Substituting (3.5), (3.6) and (3.7) in (3.4) gives the following result:

$$P(J_{ij}) = P\left(\overline{S_{i+1,i+2....j-2,j-1}}\right) - P\left(\overline{S_{i,i+1,i+2....j-2,j-1}}\right) - P\left(\overline{S_{i+1,i+2....j-2,j-1,j}}\right) + P\left(\overline{S_{i,i+1,i+2....j-2,j-1,j}}\right) \qquad (3.8)\ [8]$$

For the case of incoming traffic only and a single entrance, the probability of not stopping at a number of floors can be derived as follows:

$$P\left(\overline{S_{i,i+1,i+2....j-2,j-1,j}}\right) = P\left(\overline{S_i}\right) \cdot P\left(\overline{S_{i+1}} / \overline{S_i}\right) \cdot P\left(\overline{S_{i+2}} / \overline{S_{i,i+1}}\right) .... P\left(\overline{S_j} / \overline{S_{i,i+1,i+2....j-2,j-1}}\right) \qquad (3.9)\ [8]$$

Where the expression $P\left(\overline{S_{i+2}} / \overline{S_{i,i+1}}\right)$ represents a conditional probability that stands for the probability that a stop will not take place on floor $i+2$ given that no stops have taken place on floors i and $i+1$.

Developing equation (3.9) further gives:

$$P\left(\overline{S_{i,i+1,i+2\ldots j-2,j-1,j}}\right)=\left(\left(1-\frac{U_i}{U}\right)^P\right)\cdot\left(\left(1-\frac{U_{i+1}}{U-U_i}\right)^P\right)\cdot\left(\left(1-\frac{U_{i+2}}{U-U_i-U_{i+1}}\right)^P\right)\ldots$$

$$\ldots\ldots\left(\left(1-\frac{U_j}{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}}\right)^P\right) \qquad (3.10)^{[8]}$$

Where:

U: total building population.

U_i: ith floor population.

Developing equation (3.10) further gives:

$$P\left(\overline{S_{i,i+1,i+2\ldots j-2,j-1,j}}\right)=\left(\left(\frac{U-U_i}{U}\right)^P\right)\cdot\left(\left(\frac{U-U_i-U_{i+1}}{U-U_i}\right)^P\right)\cdot\left(\left(\frac{U-U_i-U_{i+1}-U_{i+2}}{U-U_i-U_{i+1}}\right)^P\right)\cdot$$

$$\ldots\ldots\left(\left(\frac{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}-U_j}{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}}\right)^P\right) \qquad (3.11)^{[8]}$$

$$P\left(\overline{S_{i,i+1,i+2\ldots j-2,j-1,j}}\right)=\left(\begin{array}{l}\dfrac{U-U_i}{U}\cdot\dfrac{U-U_i-U_{i+1}}{U-U_i}\cdot\dfrac{U-U_i-U_{i+1}-U_{i+2}}{U-U_i-U_{i+1}}\ldots\ldots\\[2ex]\ldots\ldots\dfrac{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}-U_j}{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}}\end{array}\right)^P \qquad (3.12)^{[8]}$$

$$P\left(\overline{S_{i,i+1,i+2\ldots j-2,j-1,j}}\right)=\left(\begin{array}{l}\dfrac{U-U_i}{U}\cdot\dfrac{U-U_i-U_{i+1}}{U-U_i}\cdot\dfrac{U-U_i-U_{i+1}-U_{i+2}}{U-U_i-U_{i+1}}\ldots\ldots\\[2ex]\ldots\ldots\dfrac{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}-U_j}{U-U_i-U_{i+1}\ldots\ldots-U_{j-1}}\end{array}\right)^P \qquad (3.13)^{[8]}$$

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$$P\left(\overline{S_{i,i+1,i+2....j-2,j-1,j}}\right) = \left(\frac{U - U_i - U_{i+1}........ - U_{j-1} - U_j}{U}\right)^P$$

$$= \left(1 - \frac{U_i + U_{i+1}........ + U_{j-1} + U_j}{U}\right)^P$$

(3.14) [8]

This leads to the general formula:

$$P\left(\overline{S_{i,i+1,i+2....j-2,j-1,j}}\right) = \left(1 - \sum_{k=i}^{j}\left(\frac{U_k}{U}\right)\right)^P$$

(3.15) [8]

Substituting (3.15) in (3.8) gives equation (3.16) which performs the probabilities of elevator transitions between floor i and floor j when the current floor i is the ground floor ($i=0$):

$$P(J_{ij}) = \left(1 - \sum_{k=i+1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=i}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=i+1}^{j}\left(\frac{U_k}{U}\right)\right)^P + \left(1 - \sum_{k=i}^{j}\left(\frac{U_k}{U}\right)\right)^P$$

(3.16) [8]

But the probability that the elevator is stopping at floor i is:

$$p\left(\overline{S_i}\right) = \left[1 - \left(1 - \frac{U_i}{U}\right)^p\right]$$

(3.17) [8]

By dividing equation (3.16) by equation (3.17), we get equation (3.18) which performs the probabilities of elevator transition from floor i to floor j when the elevator is stopping at floor i :

$$p(J_{ij}) = \left[\left(1 - \sum_{k=i+1}^{j-1}\left(\frac{U_k}{U}\right)\right)^p - \left(1 - \sum_{k=i}^{j-1}\left(\frac{U_k}{U}\right)\right)^p - \left(1 - \sum_{k=i+1}^{j}\left(\frac{U_k}{U}\right)\right)^p + \left(1 - \sum_{k=i}^{j}\left(\frac{U_k}{U}\right)\right)^p\right] \Big/ \left[1 - \left(1 - \frac{U_i}{U}\right)^p\right]$$

(3.18) [8]

Where:

i: current state (floor).

j: next state (floor).

P: number of passengers.

U_k: each floor population.

U: total population.

J: journey.

3.2.1.2 The probability of elevator's transition from the main floor to any other floor

For the special case where the journey's origin is the ground floor, the probability of a journey between the ground floor (floor 0) and floor *j*, can be developed as follows using equation (3.1) and substituting 0 for the value of *i*:

$$P(J_{0j}) = P(S_0) \cdot P(S_j) \cdot P(\overline{S_{1,2,3\dots j-2,j-1}}) \qquad (3.19)^{[8]}$$

But here the probability of stopping at the ground floor is 1 as the ground floor is the only entrance and the elevator must stop at this floor in every journey to pick up the passengers. So the equation above becomes:

$$P(J_{0j}) = P(S_j) \cdot P(\overline{S_{1,2,3\dots j-2,j-1}}) = (1 - P(\overline{S_j})) \cdot P(\overline{S_{1,2,3\dots j-2,j-1}}) \qquad (3.20)^{[8]}$$

$$P(J_{0j}) = P(\overline{S_{1,2,3\dots j-2,j-1}}) - P(\overline{S_{1,2,3\dots j-1,j}}) \qquad (3.21)^{[8]}$$

Substituting (3.15) in (3.21) gives:

$$P(J_{0j}) = \left(1 - \sum_{k=1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=1}^{j}\left(\frac{U_k}{U}\right)\right)^P \qquad (3.22)^{[8]}$$

As long as the elevator is stopping at the ground floor and the elevator transition from the ground floor to floor *j* is to be found, then:

$$p\,(\bar{S}_0) = \left[1 - \left(1 - \frac{U_0}{U}\right)^p = 1\right] \qquad (3.23)^{[8]}$$

By dividing equation (3.22) by equation (3.23) we get the probabilities of elevator transitions between the ground floor and floor j when the elevator is already stopping at the ground floor:

$$P(J_{0j}) = \left(1 - \sum_{k=1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=1}^{j}\left(\frac{U_k}{U}\right)\right)^P \Big/ 1 \qquad (3.24)^{[8]}$$

By inserting the results obtained from equation (3.18) and equation (3.24) into a matrix, we have the transition matrix that represents the probabilities of elevator's transitions between any two floors; the transition matrix is shown in Table 3.1.

Table 3.1: The elevator's Transition probability matrix , G: ground floor, N: highest floor.

	G	1	2	...	...	N-1	N
G	0	$P(J_{01})$	$P(J_{02})$	...	...	...	$P(J_{0N})$
1	$P(J_{10})$	0	$P(J_{12})$	...	...	...	$P(J_{1N})$
2	$P(J_{20})$	0	0	$P(J_{23})$	...	...	$P(J_{2N})$
. . .	$P(J_{30})$ . .	0	0	0	$P(J_{34})$	...	$P(J_{3N})$
. . . .	. .	0	0	0	0	$P(J_{45})$	$P(J_{4N})$
N-1	. . .	0	0	0	0	0	$P(J_{5N})$
N	1	0	0	0	0	0	0

In the transition matrix, the rows represent the current states and the columns represent the future states. One of the conditions of using Monte Carlo Markov Chain simulation is that the transition matrix should be a square matrix and as seen from the transition matrix, that condition is obtained where the transition matrix is an $((N + 1) \times (N + 1))$ matrix, where N+1 is the total number of floors of the building. Also the Markov Chain theory states that the summation of every row in the matrix should equal unity.

As expected the diagonal values are all zeros because the elevator's probability of transition from a floor to the same floor is zero as the elevator cannot move to any floor that it is already on $[(P(J_{ii}) = 0]$. As indicated from the incoming traffic the elevator transfers passengers only in the upper direction and while it moves down it only stops at the ground floor therefore the lower triangle values in the transition matrix are all zeros except the first row. For example the

probability of elevator's movement from the second floor to the first floor equals zero $[P(J_{21}) = 0]$. While the values of the first row of the lower triangle are not zeros because this row represents the probability of the elevator returning from its position to the ground floor. Furthermore, the probability of the elevator's returning from the last floor (N) to the ground floor equals unity always, it is because at this position the elevator has no other options to move to except the ground floor.

3.2.2 Random Scenario Generation

After deriving the probability equations and arranging them in the transition matrix we can generate random scenarios for the elevator's transition depending on the probability function. The random scenario is the elevator's journey from the ground floor up to the destination floors and returning to the ground floor. The selection process of the destination floors depends on the transition matrix.

Therefore we shall sketch the Probability Density Function (PDF) and the Cumulative Distribution Function (CDF) for every current floor recalling that rows of the transition matrix corresponds to current floors, i.e. first row corresponds to first floor and second row corresponds to second floor,…etc.

Probability Density Function (PDF) is a function representing the relative distribution of frequency of the elevator's transition between two floors, see figure 3.1.The probability density function is nonnegative everywhere, and its integral over the entire space is equal to one. PDF has the property that its integral from a to b is the probability that the variable lies in this interval, hence we also sketch the cumulative density function. Cumulative Distribution Function (CDF) is the sum or integral function of the probability density function, see figure 3.2.
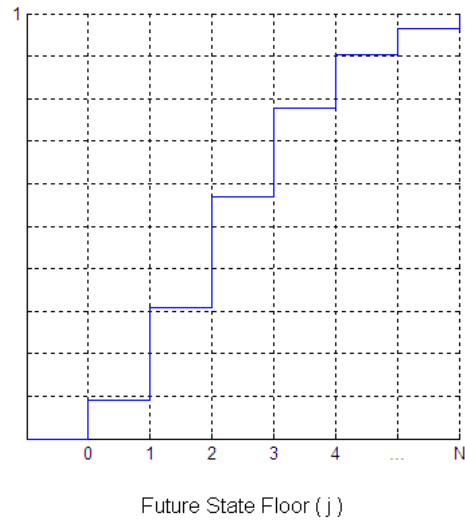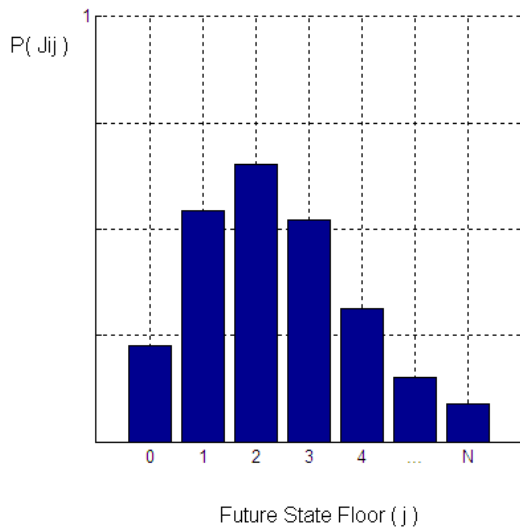
Figure 3.1: PDF graph for the current floor i.   Figure 3.2: CDF graph for the current floor i

We start the elevator's journey at the ground floor as the passengers enter the elevator only at the ground floor and then we take a random value between (0-1) and scan the CDF of the ground floor to find which interval this number belongs to; hence we determine the destination floor (future state). In the next step the previous future state floor becomes the current state floor so we take another random value between (0-1) and scan the CDF of the corresponding current floor and find the interval that this random value belongs to, to determine the future state floor. We continue this procedure till we return to the ground floor again; and at the end of this procedure we will have one random journey scenario.

3.2.3 Calculation of the kinematics time

The kinematics time is the time needed for the elevator to move between floors in the whole journey; i.e. to move from the ground floor up to the destination floors and then return from the highest reversal floor to the ground. To calculate the kinematics time we shall calculate the transition time for each transition between two floors, see figure 3.3.
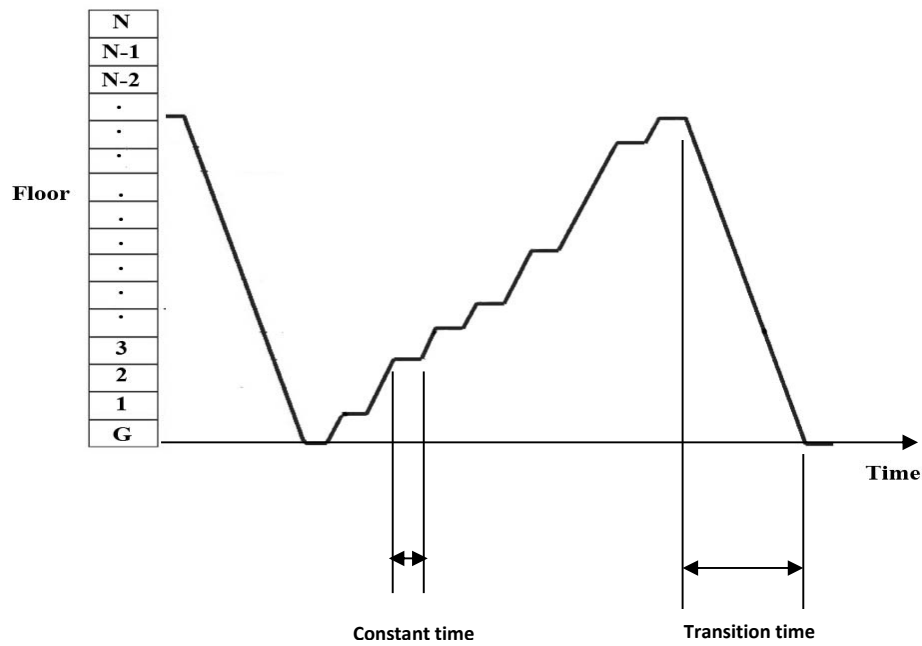
Figure 3.3: Transition time and constant time

The transition time is calculated using one of these equations:

$$\text{If } d \geq \left(\frac{a^2 v + v^2 j}{aj} \right) \text{ then } t = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} \tag{3.25}$$

$$\text{If } \frac{2a^3}{j^2} \leq d < \left(\frac{a^2 v + v^2 j}{aj} \right) \text{ then } t = \frac{a}{j} + \sqrt{\frac{4d}{a} + \left(\frac{a}{j} \right)^2} \tag{3.26}$$

$$\text{If } d < \frac{2a^3}{j^2} \text{ then } t = \left(\frac{32d}{j} \right)^{\frac{1}{3}} \tag{3.27}$$

Where :

d: distance traveled [m]

v: velocity [m/s]

a: acceleration [m/s^2]

j: Jerk [m/s^3]

And because we have assumed that top speed is attained in one floor journey; i.e. $d_f \geq \left(\dfrac{a^2v + v^2 j}{aj} \right)$, then we use the first formula.

After calculating the transition times from the moment the elevator moves from the ground floor till it returns to it, the transition times should be summed to give the total kinematics time.

$$\tau_k = \sum t_{ij} \tag{3.28}$$

Where:

τ_k: Kinematics time.

t_{ij}: Time needed for the elevator's transition between two floors.

3.2.4 Calculation of the constant time

The constant time represents the time spent by the elevator while it is stopped. It includes the time needed for the elevator's door to open and close and the time needed for the passengers to alight and board. It is calculated using this formula.

$$\tau_c = S \left(t_{do} + t_{dc} \right) + P \left(t_{pi} + t_{po} \right) \tag{3.29}$$

Where:

t_{do}: Door opening time. [s]

t_{dc}: Door closing time. [s]

t_{pi}: Passenger's boarding time. [s]

t_{po} : Passenger's alighting time. [s]

P : number of passengers boarding the car.

S: number of stops in one journey.

Since S is the number of times the elevator stops in each journey, the value of S equals the number of random numbers we use to complete a whole journey.

3.2.5 Calculation of the round trip time

The summation of the kinematics time and the constant time gives the round trip time.

$$\tau = \tau_k + \tau_c \tag{3.30}$$

3.2.6 Trials of the procedure

To obtain a RTT value that is very close to the actual value we iterate this procedure for a very large number of times and find the average of the RTTs found through this formula.

$$\tau_{avg} = \frac{\tau_1 + \tau_2 + \cdots + \tau_n}{n} \tag{3.31}$$

Where:

n: is the number of trials

τ_n : the round trip time found in the nth trial

The accuracy of the Monte Carlo Markov chain simulation method depends on the number of trials. As the number of trials increases, the accuracy of the procedure increases and the round trip time calculated becomes closer to the exact value which therefore minimizes the percentage of error.

3.3 Monte Carlo Simulation method

The Monte Carlo method is quite similar to Monte Carlo Markov Chain method where both are based on the generation of random journey scenarios. In the Monte Carlo Markov chains method the random scenario generation was based on the transition probability matrix but in the Monte Carlo method the random scenario generation will be based on both the percentage of floor population and the number of passengers inside the car[9]. On the other hand, the two methods differ where in the Monte Carlo Markov Chain method the number of stops was randomly found and varies from a scenario to another. However, in the Monte Carlo method the number of stops is constant.

The number of passengers is found as the effective capacity of the car based on 80% car-rated capacity:

$$P = 0.8 \cdot CC \qquad\qquad (3.32)\,[9]$$

Where

CC: car capacity.

The Monte Carlo simulation procedure includes four main steps;

1- Drawing the PDF, CDF graphs of the floor population percentage.
2- Generation of the random journey scenario.
3- Calculation of the RTT.
4- Trials of the procedure.

3.3.1 Drawing the PDF and CDF graphs of the floor population percentage

The probability of elevator's transition between any two floors depends on the population distribution upon floors. By normalization of each floor population we can calculate the floor population percentage. Normalization of the floor population is shown in Table 3.2.

Table 3.2: Normalization of the floor population, where U is the total population.

Floor k	Floor population	Percentage of floor population
1	u_1	u_1/U
2	u_2	u_2/U
. . .	. . .	. . .
N-1	u_{N-1}	u_{N-1}/U
N	u_N	u_N/U

Thus, the floor population percentage equals:

$$[u_1/U \quad u_2/U \quad \dots \quad u_{N-1}/U \quad u_N/U]$$

But in our case, we have assumed equal floor population, which means that the floor population percentage will be equal.

After calculating the population percentage and calculating the number of passengers, PDF and CDF graphs of the floor population percentage are plotted, see figure3.4 and figure3.5.
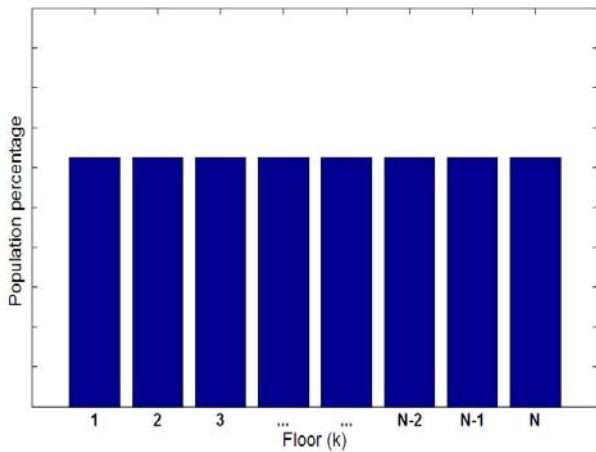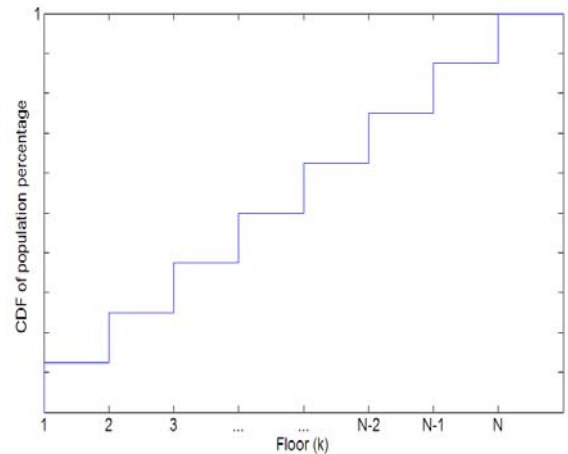

Figure 3.4: PDF graph for the current floor *i*.

Figure 3.5: CDF graph for the current floor *i*

3.3.2 Generation of the random journey scenario

Random numbers are all generated at once, it is necessary to generate (*P*) number of random values for each scenario, and then these random numbers is taken and the CDF is being scanned to find to which interval these numbers belong to, and thus the upper destination floors for all passengers are generated. Then, as the car becomes empty, the elevator will return to the ground floor.

3.3.3 Calculation of the round trip time

The procedure of calculating the value of the round trip time is the same as the procedure previously explained in the Monte Carlo Markov chain simulation method, where the summation of the kinematics time and constant time gives the round trip time of the random journey scenario.

3.3.4 Trials of the procedure

After calculating the round trip time of the random journey scenario this procedure must be repeated for a very large number of times, while the round trip time is found in each time. At the

end the average of the round trip times is found to get the round trip time of the system that will be close to the exact value.

As the number of trials increases, the accuracy of this method increases. But, it is important to mention that the Monte Carlo simulation method needs more trials to get close to the round trip time exact value than the trials used in the Monte Carlo Markov chain simulation method.

3.4   The analytical solution

In this method the number of stops, the highest reversal floor and the number of passengers are found using some equations and are taken as the average number that occurs in one trip journey.

The average number of stops in one trip journey is called the probable number of stops (S). For equal floor population, the probable number of stops is calculated through this formula:

$$S = N\left(1 - \left(1 - \frac{1}{N} \right)^P \right)$$

(3.33) [10]

Where:

N: the total number of floors.

P: the number of passengers.

The number of passengers (P) is taken as the effective capacity of the car based on 80% car-rated capacity, which is found in equation (15).

As already mentioned before, the highest reversal floor is the highest floor that the elevator reaches in one trip journey. For equal floor population the highest reversal floor is found using this formula:

$$H = N - \sum_{i=1}^{N-1} \left(\frac{i}{N} \right)^P$$

(3.34) [10]

S, P and H do not need to be integers. These three parameters are then substituted in the RTT general equation. The round trip time is made up of three main components; the time spent at the ground floor collecting the passengers, the time needed for the elevator to travel to the upper

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floors and deliver the passengers to their destinations, and the time needed for the elevator to return from the highest reversal floor to the ground floor, this is expressed in equation (3.35).

$$\tau = \tau_G + \tau_S + \tau_H \qquad \text{(3.35)} \, ^{[10]}$$

Where:

τ_G: The time spent at the ground floor.

τ_S : The time spent travelling to the upper destination floors and delivering the passengers.

τ_H : The time spent returning back to the ground terminal from the highest reversal floor.

The time spent at the ground floor consists of the door opening time, the time needed for the passengers to enter the elevator, the door closing time and the motor starting delay minus the advanced door opening time, Therefore:

$$\tau_G = \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\!\left(t_{pi}\right) \qquad \text{(3.36)} \, ^{[10]}$$

Where:

t_{do} : Door opening time

t_{dc}: Door closing time

t_{sd}:Motor start delay

t_{ao}: Advance door opening time

t_{pi}: Passenger boarding time

P: number of passengers

The time spent delivering the passengers to their destinations consist of two parts; the kinematics time and constant time. The kinematics time contains the time of transition from the ground floor to the highest reversal floor, while the constant time is the time caused by all the stops of the elevator. Each stop causes the elevator to decelerate, open its door, alight passengers, close its door and then accelerates again, therefore we have this formula:

$$\tau_S = \left(S \cdot \left(t_{acc} + t_{dec}\right) + \left(\frac{H \cdot d_f}{v}\right) \right) + S \cdot \left(t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\!\left(t_{po}\right) \qquad \text{(3.37)} \, ^{[10]}$$

Where:

S: probable number of stops.

H: highest reversal floor.

d_f: floor height.

t_{acc} : the time taken to accelerate up to the top speed from standstill.

t_{dec} : the time taken to decelerate down from the top speed down to standstill.

v : the rated speed.

t_{po}: Passenger alighting time.

Rewriting the equation gives:

$$\tau_S = H \cdot \left(\frac{d_f}{v}\right) + S \cdot \left(t_{acc} + t_{dec} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\left(t_{po}\right)$$

[10]

(3.38)

Since we have assumed that the top speed is attained in one floor journey, then:

$$t_{acc} + t_{dec} = \frac{v}{a} + \frac{a}{j}$$

(3.39)

Therefore equation (3.35) becomes:

$$\tau_S = H \cdot \left(\frac{d_f}{v}\right) + S \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\left(t_{po}\right)$$

(3.40) [10]

The third component of the round trip time is the time needed for the elevator to travel from the highest reversal floor to the ground floor, therefore:

$$\tau_H = \left(\frac{H \cdot d_f}{v} + \frac{v}{a} + \frac{a}{j}\right)$$

(3.41) [10]

The summation of these three components gives the round trip time:

$$\tau = 2 \cdot H \cdot \left(\frac{d_f}{v}\right) + (S+1) \cdot \left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P\left(t_{pi} + t_{po}\right) + 2 \cdot \left(\frac{d_G - d_f}{v}\right)$$

(3.42)[10]

The analytical solution method is very accurate and easy. But, it only exists for the general four assumptions or can be derived in the absence of one of these assumptions. However, if a combination of these assumptions were absent, it is very difficult to derive analytical equations for these cases as the problem becomes very complicated.

Chapter four

Software Simulation

4.1 Introduction

The use of Monte Carlo Markov Chain simulation has become practical only with the availability of powerful and fast computing programs that are readily accessible within desktops and laptops. This makes it easy to run tens of thousands of simulation runs in a fraction of a second.

In this section, I will discuss the software implementation of the Monte Carlo Markov Chain simulation to calculate the elevator's round trip time during up peak traffic conditions. MATLAB GUI was used in the software implementation. The software tool finds the round trip time using the Monte Carlo Markov chain method as well as Monte Carlo method and the analytical method.

4.2 The MATLAB GUI round trip time simulation tool

MATLAB GUI refers to Graphical User Interface and it is a MATLAB tool that allows users to perform tasks interactively through controls such as buttons and sliders. The MATLAB GUI allows the user to enter the building data and obtain the value of the round trip time using any of the three methods discussed before, with different levels of certainty chosen by the user by choosing the number of trials. The MATLAB GUI round trip time simulation tool is shown below in figure 4.1.
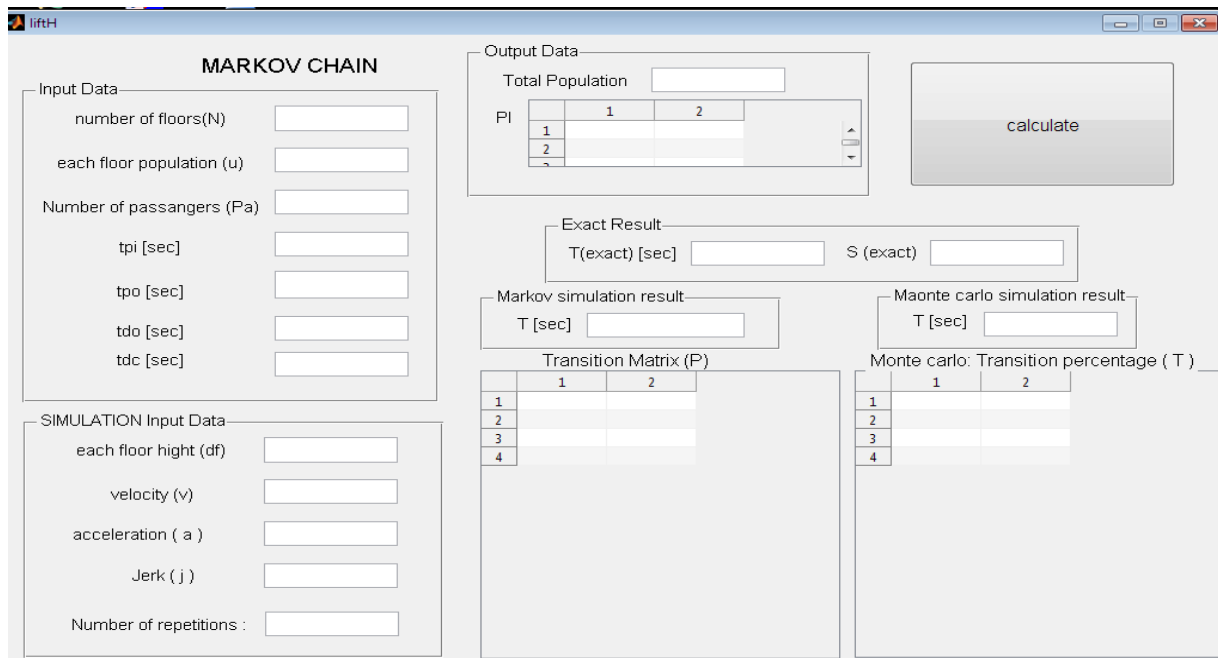

Figure 4.1: The MATLAB GUI round trip time simulation tool

The inputs of the MATLAB GUI round trip time simulation tool are:

1- The number of floors.
2- Floor population.
3- Number of passengers.
4- Floor height.
5- Speed of the elevator motor.
6- Acceleration of the elevator motor.
7- Jerk of the elevator motor.
8- Time needed for passengers to board and alight.
9- Time needed for car door to open and close.
10- Number of trials desired.

The outputs of the MATLAB GUI round trip time simulation tool are:

1- The round trip time value found using Monte Carlo Markov chain simulation.
2- The round trip time value found using Monte Carlo simulation.
3- The analytical round trip time value.
4- The transition probability matrix.
5- The transition percentage matrix "from Monte Carlo simulation results'.

The Monte Carlo Markov Chain simulation tool contains five main blocks;
1- The transition probability matrix generator.
2- The random journey scenario generator.
3- The kinematics time calculator.
4- The constant time calculator.
5- The round trip time calculator.

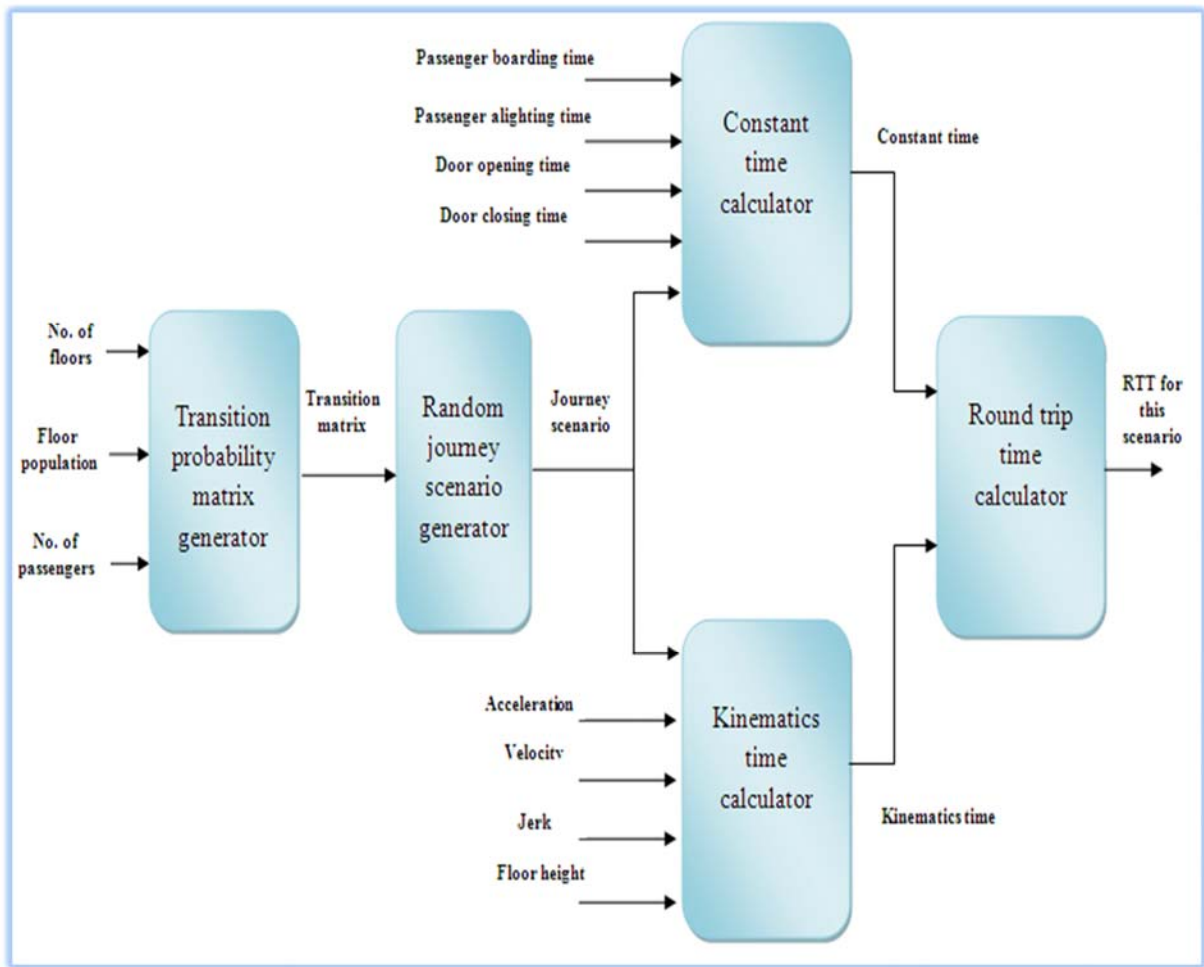The block diagram of the Monte Carlo Markov Chain simulation round trip time tool is shown in figure 4.2 below.

Figure 4.2: Block diagram of the Monte Carlo Markov Chain round trip time tool

The Monte Carlo simulation tool contains five main blocks;

1- The random journey scenario generator.
2- Kinematics time calculator.
3- The constant time calculator.
4- The round trip time calculator.

The block diagram of the Monte Carlo simulation round trip time tool is shown in figure 4.3 below.
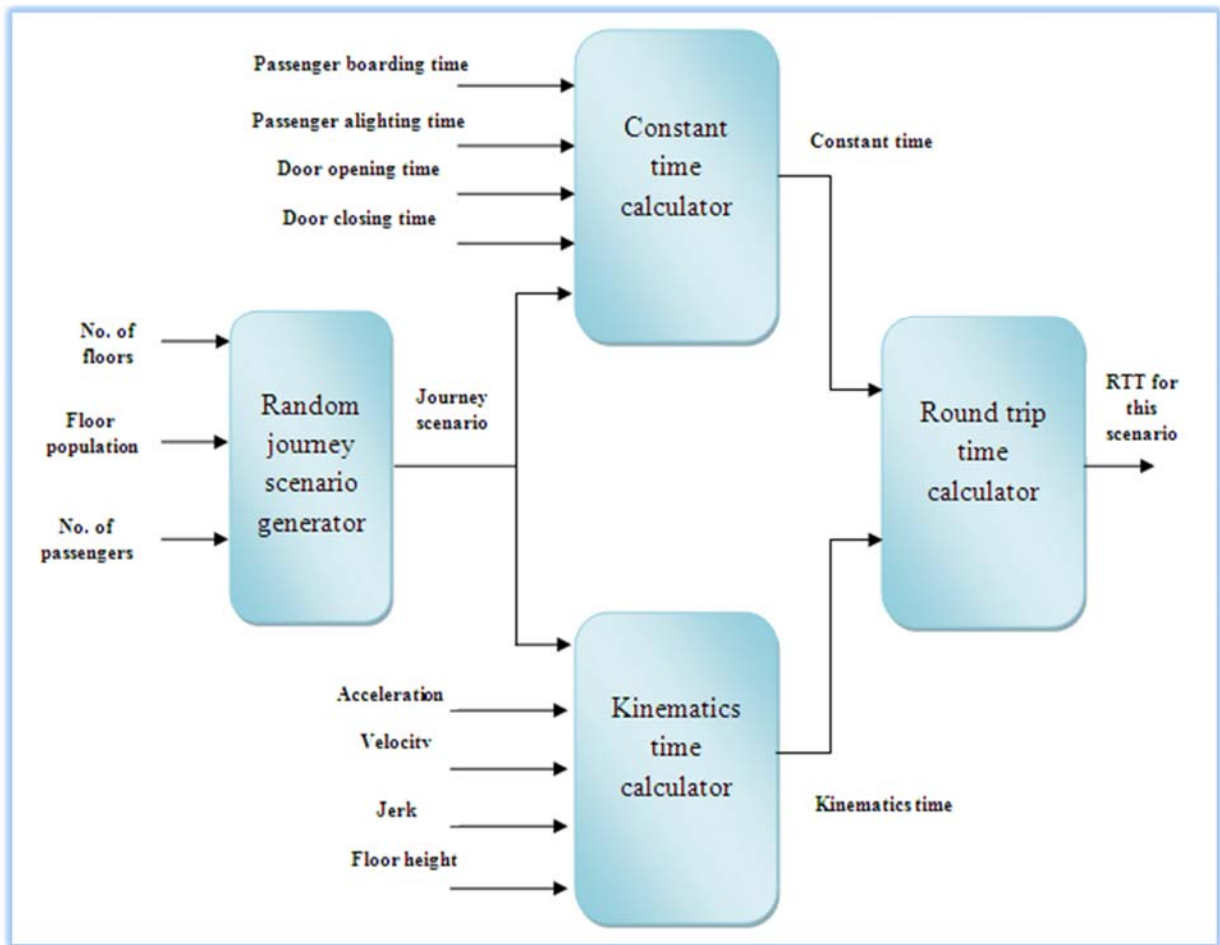
Figure 4.3: Block diagram of the Monte Carlo round trip time simulation tool

The Monte Carlo simulation tool also produces the transition percentage from the Monte Carlo simulation to compare it with the transition probability matrix used in Monte Carlo Markov chain method. Results show that this transition percentage matrix is close to the transition probabilities matrix of the Monte Carlo Markov chain method.

The simulation tool also calculates the round trip time using the analytical method, in order to compare the results found using Monte Carlo Markov chain simulation method and Monte Carlo simulation method with the analytical value.

Chapter Five

Results, Conclusions and recommendations

5.1 Case study 1

An example from a building is used to illustrate the use of Monte Carlo Markov chain simulation. The parameters for a typical building with equal floor heights and equal floor population are shown below.

1. Number of floors above the ground floor equals 5.
2. Car capacity is 6 persons.
3. Finished floor level to finished floor level (floor height) is 4.5 m.
4. Each floor population is 100 persons.
5. Rated speed is 1.6 m.s^{-1}.
6. Rated acceleration is 1 m.s^{-2}.
7. Rated jerk is 1 m.s^{-3}.
8. Passenger alighting time equals 1.2 s.
9. Passenger boarding time equals 1.2 s.
10. Door opening time equals 2 s.
11. Door closing time equals 3 s.
12. Start delay is 0.
13. Advanced opening time is 0.

We will start with the exact solution; a check needs to be carried out to ensure that the elevator will attain top speed in a one floor journey, as follows:

$$d_f = 4.5 \geq \left(\frac{a^2\,v\,+\,v^2\,j}{a\,j}\right) = \left(\frac{1\,(1.6)\,+\,1.6^2\,(1)}{1}\right) = 4.16\,m$$

So the top speed of 1.6 m/s is attained in a one-floor journey. In this case the time taken to complete a one-floor jump, t_f will be:

$$t_f = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.4\,s$$

The value of P based on 80% car-rated capacity is:

$$P = 0.8\,CC = 0.8\,(6) = 4.8\,passengers$$

The values of the probable number of stops (S), highest reversal floor (H) are as follows:

$$S = N\left(1 - \left(1 - \frac{1}{N}\right)^P\right) = 5\left(1 - \left(1 - \frac{1}{5}\right)^{4.8}\right) = 3.29$$

$$H = N - \sum_{i=1}^{N-1}\left(\frac{i}{N}\right)^P = 5 - \sum_{i=1}^{4}\left(\frac{i}{5}\right)^{4.8} = 4.56$$

Substituting in the round trip time equation gives:

$$\tau = 2H\left(\frac{d_f}{v}\right) + (s+1).\left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{pi} + t_{po})$$

$$= 2\,(4.56)\left(\frac{4.5}{1.6}\right) + (3.29+1)\left(5.4 - \frac{4.5}{1.6} + 2 + 3 + 0 - 0\right) + 4.8(1.2 + 1.2) = 69.74\ s$$

So, the value of the round trip time found using the exact value method equals 69.74 s.

Using the Monte Carlo method, the number of passengers P equals the 80% car-rated capacity, hence P= 4.8.

Then the PDF, CDF graphs for the floor population percentage should be sketched. As the floor population is equal for all floors then $U = N\,(100) = 100\,(5) = 500\ persons$

$U_k/U = \frac{100}{500} = \frac{1}{5} = 0.2$, where k indicates floors from 1 to N.

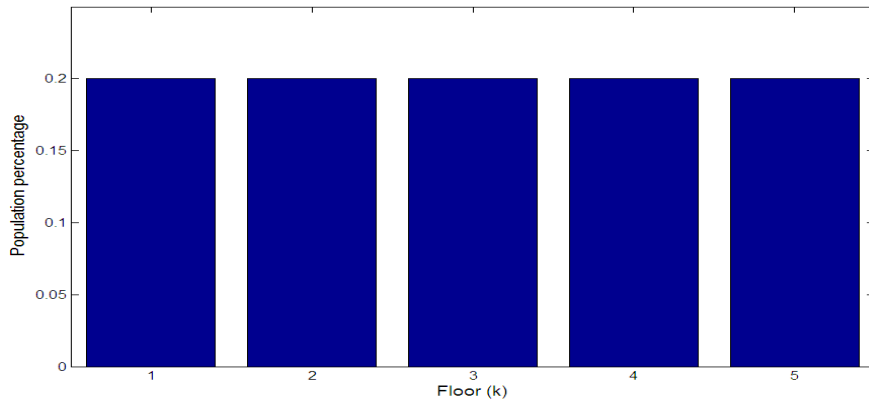The PDF and CDF graphs are sketched as shown in figure 5.1 and figure 5.2.

Figure 5.1: the probability density function of the floors population percentage
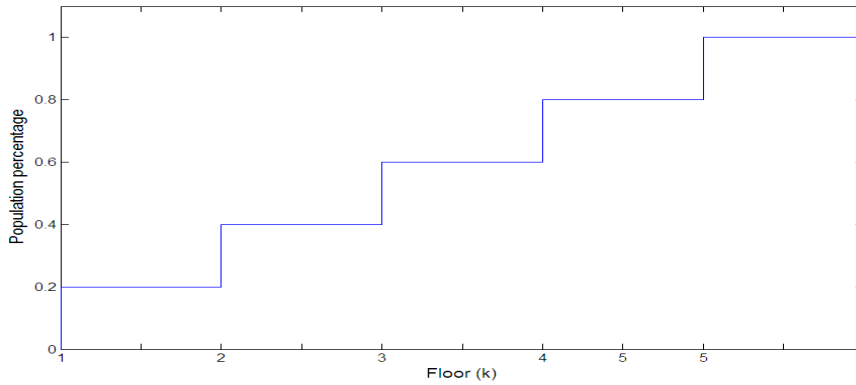
Figure 5.2: The cumulative density function of the floors population percentage

The next step is to generate five random numbers between (0-1), the random numbers generated are: 0.391, 0.936, 0.12, 0.317, 0.081

By scanning the CDF, the random generated floors based on the floor population percentage are: First floor, fifth floor, first floor, second floor and first floor. So there will be one stop on the first floor, one stop on the second floor and one stop on the fifth floor. Therefore, the number of stops equals three.

The kinematics time for this journey scenario is:

$$\tau_k = \tau_{01} + \tau_{12} + \tau_{25} + \tau_{50}$$

Where:

τ_{01} : is the transition time between the ground floor and the first floor.

τ_{12}: is the transition time between the first floor and the second floor.

τ_{25}: is the transition time between the second floor and the fifth floor.

τ_{50}: is the transition time between the fifth floor and the ground floor.

Since the top speed is attained in one floor journey the transition times are calculated as follows:

$$\tau_{01} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41\ s$$

$$\tau_{12} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41\ s$$

$$\tau_{25} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5\ (3)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 11.04\ s$$

$$\tau_{50} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5\,(5)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 16.66\ s$$

The kinematics time equals to:

$$\tau_k = \tau_{01} + \tau_{12} + \tau_{25} + \tau_{51} = 5.41 + 5.41 + 11.04 + 16.66 = 38.52\ s$$

The constant time for this journey scenario is calculated as follows:

$$\tau_c = S\,(t_{do} + t_{dc}) + P\,(t_{pi} + t_{po}) = 3(2 + 3) + 4.8(1.2 + 1.2) = 26.52\ s$$

The round trip time for one trial equal:

$$\tau = \tau_k + \tau_c = 21.86 + 26.52 = 65.04\ s$$

Using the Monte Carlo software tool, the round trip time found for one trial equal: 51.195 s

It is important to recall that the round trip time value found for one trial differ from trial to another as the procedure is totally random and depends on the set of random numbers generated each time.

Using the Monte Carlo Markov chain method, first we shall calculate the transition matrix by calculating the probabilities of transitions using these two equations:

The probability of elevator transition between any two floors:

$$p\,(J_{ij}) = \left[\left(1 - \sum_{k=i+1}^{j-1}\left(\frac{U_k}{U}\right)\right)^p - \left(1 - \sum_{k=i}^{j-1}\left(\frac{U_k}{U}\right)\right)^p - \left(1 - \sum_{k=i+1}^{j}\left(\frac{U_k}{U}\right)\right)^p + \left(1 - \sum_{k=i}^{j}\left(\frac{U_k}{U}\right)\right)^p\right] \Big/ \left[1 - \left(1 - \frac{U_k}{U}\right)^p\right]$$

The probability of elevator transition between the main floor and any other floor:

$$P(J_{0j}) = \left(1 - \sum_{k=1}^{j-1}\left(\frac{U_k}{U}\right)\right)^P - \left(1 - \sum_{k=1}^{j}\left(\frac{U_k}{U}\right)\right)^P \Big/ 1$$

The transition matrix is shown in table 5.1 below.

Table5.1: the transition probability matrix of case study 1

	0	1	2	3	4	5
0	0	0.6574	0.2565	0.0738	0.0119	4.4151e-04
1	6.7164e-04	0	0.6098	0.2779	0.0943	0.0174
2	0.0180	0	0	0.6098	0.2779	0.0943
3	0.1123	0	0	0	0.6098	0.2779
4	0.3902	0	0	0	0	0.6098
5	1	0	0	0	0	0

After that, PDF and CDF graphs should be sketched for each row of the matrix, they are shown in figures below (figures 5.3- 5.14).
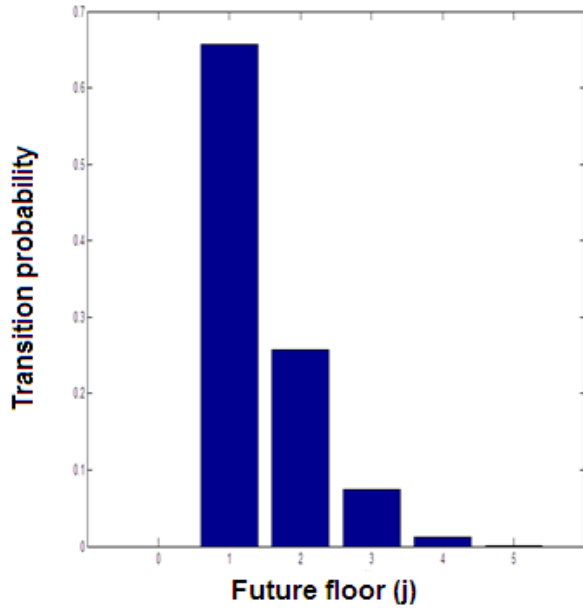

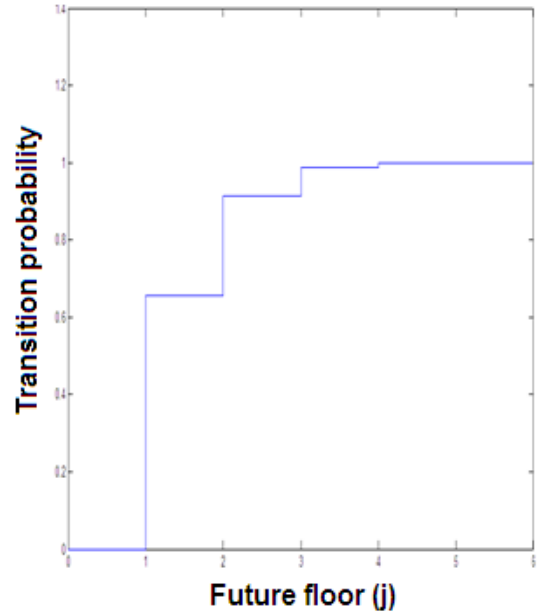Figure 5.3: PDF of the ground floor
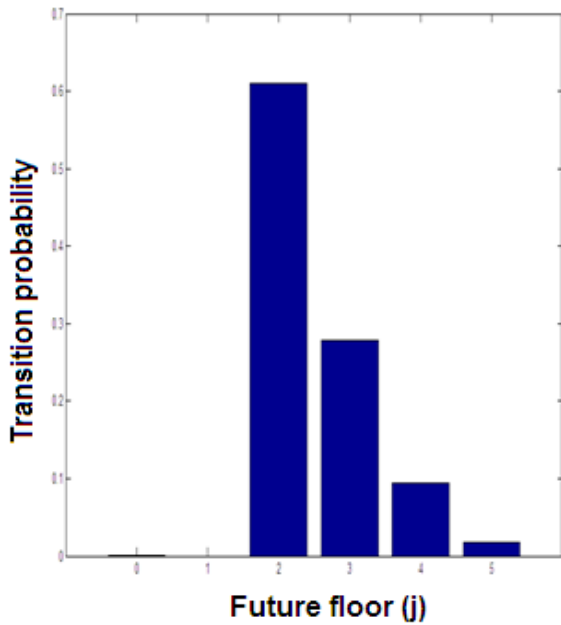

Figure 5.4: CDF of the ground floor

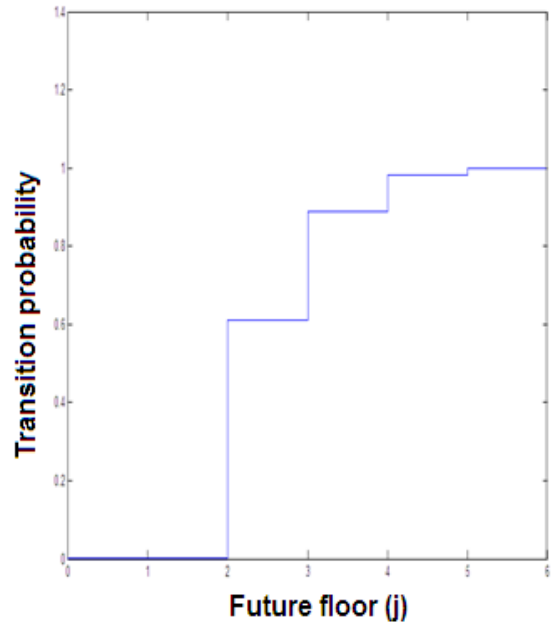Figure 5.5: PDF of the first floor

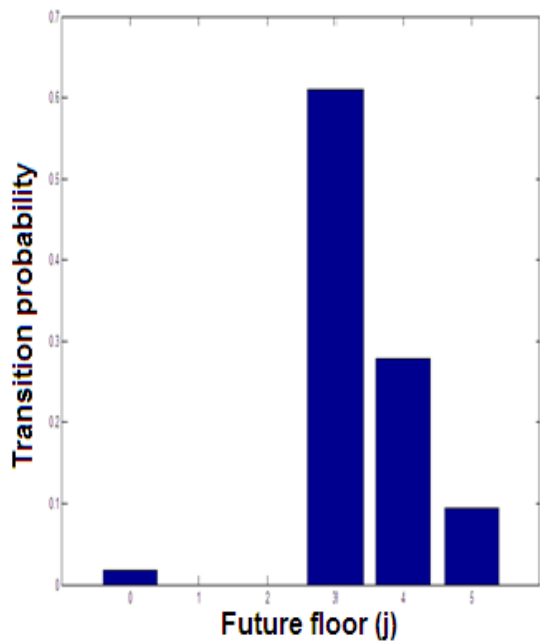Figure 5.6: CDF of the first floor

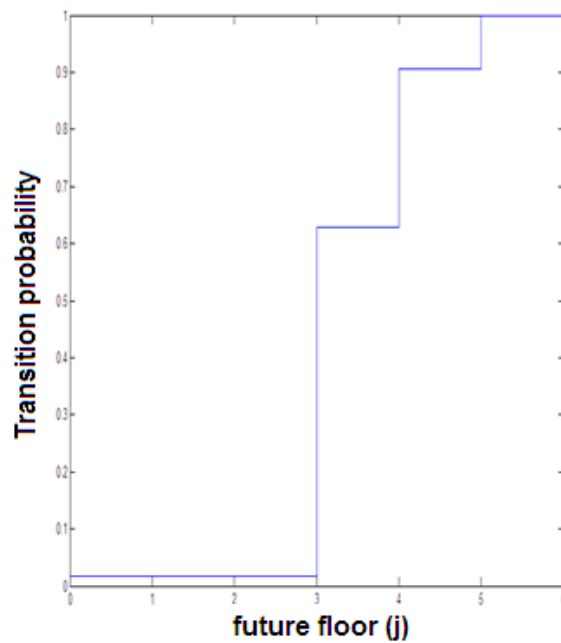Figure 5.7: PDF of the second floor
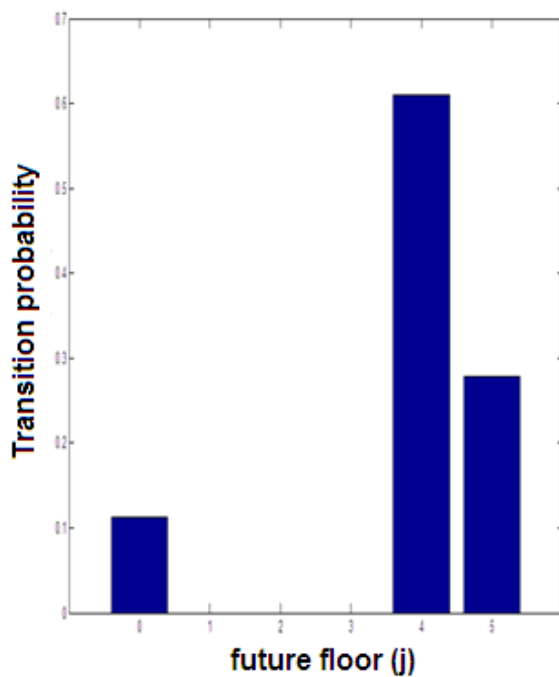

Figure 5.8: CDF of the second floor

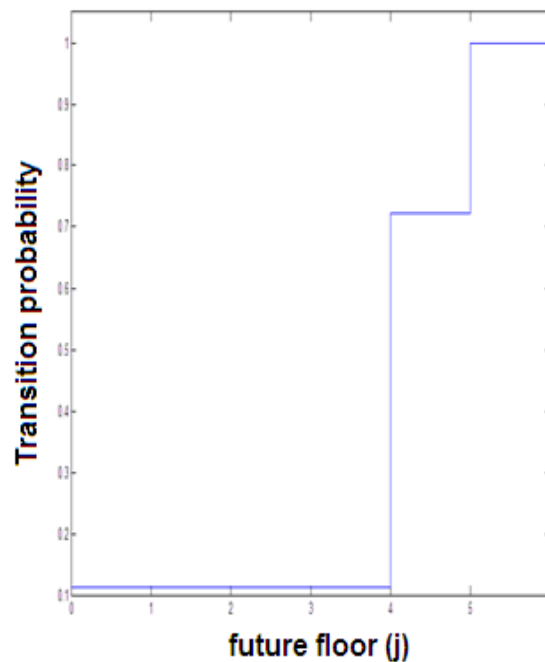Figure 5.9: PDF of the third floor

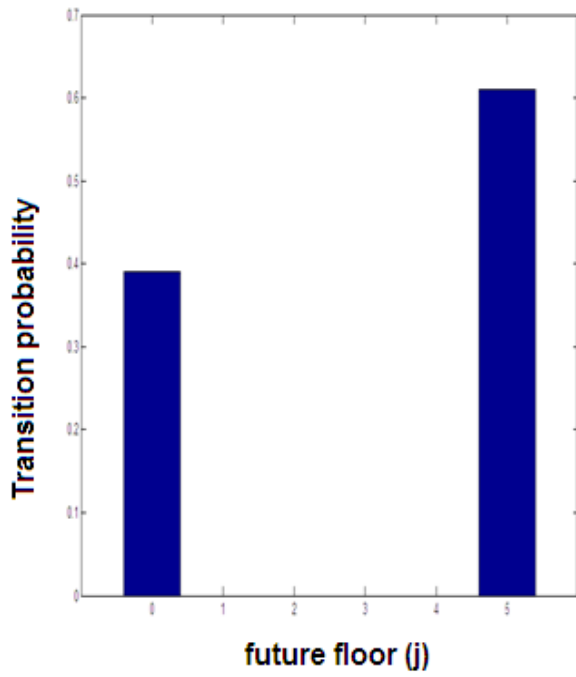Figure 5.10: CDF of the third floor
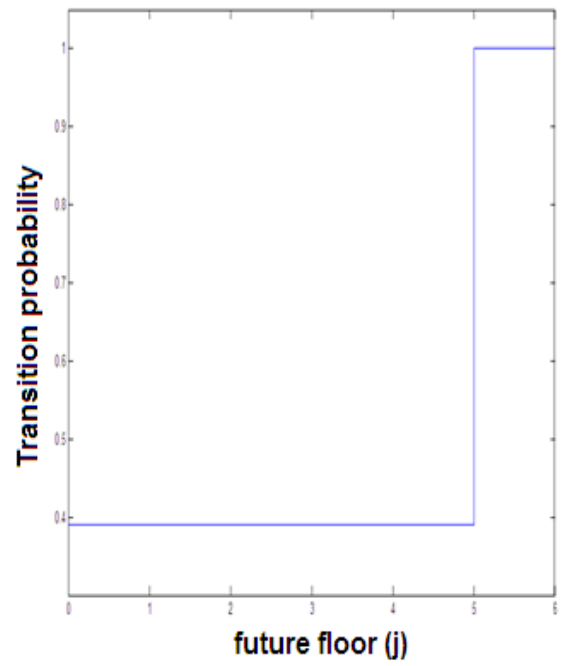
Figure 5.11: PDF of the fourth floor
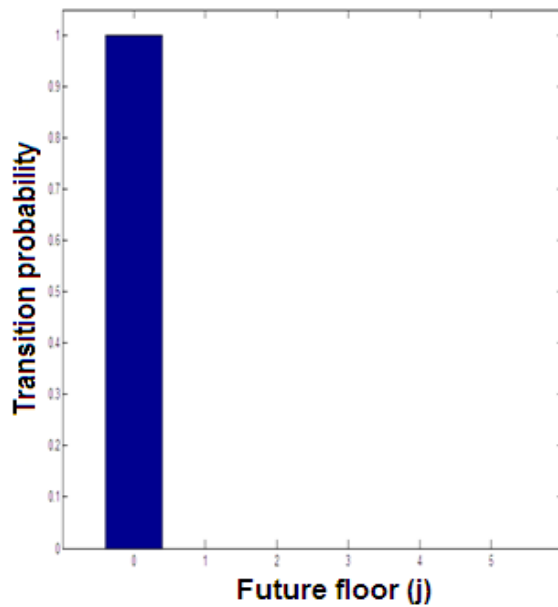

Figure 5.12: CDF of the fourth floor

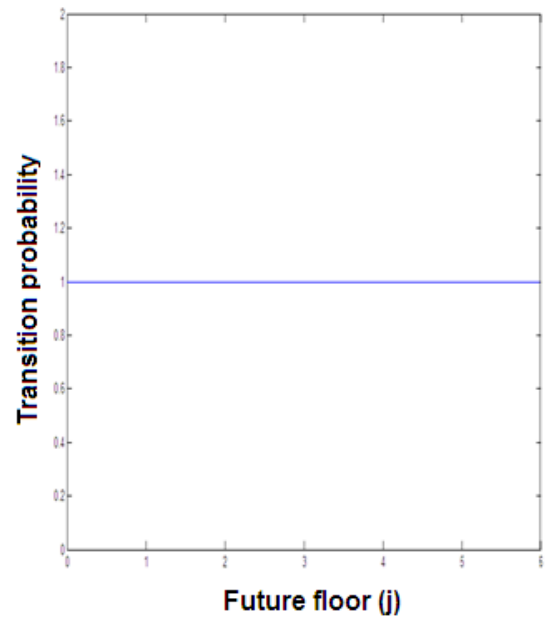Figure 5.13: PDF of the fifth floor

Figure 5.14: CDF of the fifth floor

Afterwards, the first random number is generated, the first random number is 0.869. By scanning the ground floor's CDF we have the first random destination which is the second floor. Taking another random number, we have 0.935, by scanning the second floor's CDF we have the next destination which is in this case is the fourth floor. Taking another random number we have 0.995 by scanning the fourth floor's CDF we get the next destination which in this case is the fifth floor. Taking another random number we have 0.691, by scanning the fifth floor CDF we have the next destination which is the ground floor.

Now, the first journey scenario is generated; where the elevator will stop first at the second floor then at the fourth floor then at the fifth floor before it turns back to the ground floor, hence we number of stops for this journey equals three. Number of passengers equals three too.

The kinematics time for this journey scenario is:

$$\tau_k = \tau_{02} + \tau_{24} + \tau_{45} + \tau_{50}$$

Where:

τ_{02} : is the transition time between the ground floor and the second floor.

τ_{24}: is the transition time between the second floor and the fourth floor.

τ_{45}: is the transition time between the fourth floor and the fifth floor.

τ_{50}: is the transition time between the fifth floor and the ground floor.

Since the top speed is attained in one floor journey the transition times are calculated as follows:

$$\tau_{02} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(2)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 8.225 \ s$$

$$\tau_{24} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(2)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 8.225 \ s$$

$$\tau_{45} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(1)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41 \ s$$

$$\tau_{50} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(5)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 16.66 \ s$$

The kinematics time equals to:

$$\tau_k = 8.225 + 8.225 + 5.41 + 16.66 = 38.52 \ s$$

The constant time for this journey scenario is calculated as follows:

$$\tau_c = S \ (t_{do} + \ t_{dc}) + P \ (t_{pi} + t_{po}) = 3(2 + 3) + 3(1.2 + 1.2) = 22.2 \ s$$

The round trip time for one trial equal:

$$\tau = \tau_k + \tau_c = 38.52 + 22.2 = 60.72 \ s$$

Using the Monte Carlo Markov chain software tool, the round trip time found for one trial equal: 62.445 s. It is important to recall that the round trip time value found for one trial differ from trial to another as the procedure is totally random and depends on the set of random numbers generated each time.

Using the simulation software, we will get the results shown in the table 5.2 below.

Table 5.2: the simulation software results for the values of round trip time using MCMC and MC methods for different number of trials

Number of Trials	RTT using MCMC	RTT using Monte Carlo
100	69.6813	65.4935
1000	69.7998	66.1725
10000	69.7188	67.6293
100000	69.736	68.9655

After that, the percentage of error for Monte Carlo Markov Chain method is calculated as follows:

$$e\% = \left| \frac{exact\ value - MCMC\ value}{exact\ value} \right| \times 100\%$$

Likewise, the percentage of error for Monte Carlo method is calculated as follows:

$$e\% = \left| \frac{exact\ value - MC\ value}{exact\ value} \right| \times 100\%$$

The error percentage values are calculated and filled in table 5.3 below.

Table 5.3: percentage of error for Monte Carlo Markov chain method and Monte Carlo method for case study 1

Number of Trials	MCMC percentage of error	Monte Carlo percentage of error
100	0.086%	6.091%
1000	0.084%	5.117%
10000	0.032%	3.028%
100000	7.743×10^{-3}%	1.112%

The percentage of error functions is sketched and shown in figure 5.15 below.
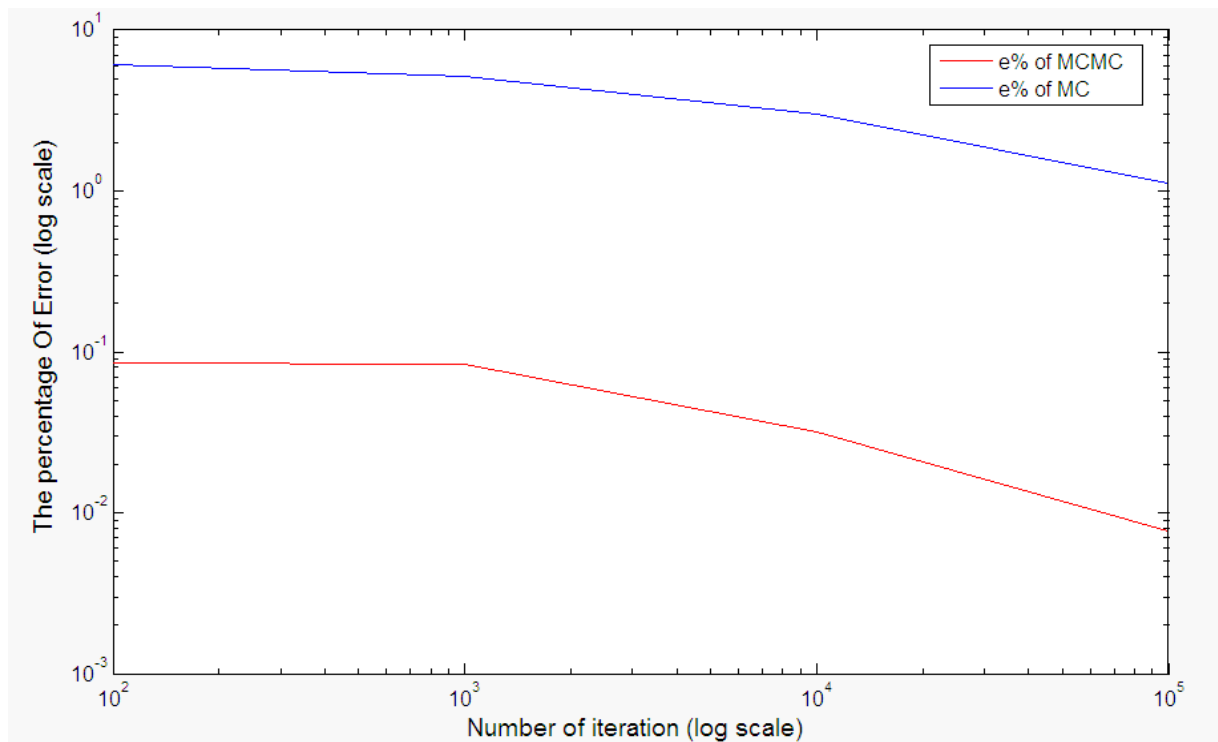

Figure 5.15 : Percentage of error versus number of trials in log scale

5.2 Case study 2

Another example from a building is used to illustrate the use of Monte Carlo Markov chain simulation. The parameters for a typical building with equal floor heights and equal floor population are shown below.

1- Number of floors above the ground floor equals 10.
2- Car capacity is 8 persons.
3- Finished floor level to finished floor level (floor height) is 4.5 m.
4- Each floor population is 150 persons.
5- Rated speed is 1.6 m.s^{-1}.
6- Rated acceleration is 1 m.s^{-2}.
7- Rated jerk is 1 m.s^{-3}.
8- Passenger alighting time equals 1.2 s.
9- Passenger boarding time equals 1.2 s.
10- Door opening time equals 2 s.
11- Door closing time equals 3 s.
12- Start delay is 0.
13- Advanced opening time is 0.

We will start with the exact solution; a check needs to be carried out to ensure that the elevator will attain top speed in a one floor journey, as follows:

$$d_f = 4.5 \geq \left(\frac{a^2\, v\, +\, v^2\, j}{a\, j} \right) = \left(\frac{1\,(1.6)\, +\, 1.6^2\,(1)}{1} \right) = 4.16\, m$$

So the top speed of 1.6 m/s is attained in a one-floor journey. In this case the time taken to complete a one-floor journey, t_f will be:

$$t_f = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.4\, s$$

The value of P based on 80% car-rated capacity is:

$$P = 0.8\, CC = 0.8\,(8) = 6.4\, passengers$$

The values of the probable number of stops (S), highest reversal floor (H) are as follows:

$$S = N\left(1 - \left(1 - \frac{1}{N}\right)^P\right) = 10\left(1 - \left(1 - \frac{1}{10}\right)^{6.4}\right) = 4.905$$

$$H = N - \sum_{i=1}^{N-1}\left(\frac{i}{N}\right)^P = 10 - \sum_{i=1}^{9}\left(\frac{i}{10}\right)^{6.4} = 9.096$$

Substituting in the round trip time equation gives:

$$\tau = 2H\left(\frac{d_f}{v}\right) + (s+1).\left(t_f - \frac{d_f}{v} + t_{do} + t_{dc} + t_{sd} - t_{ao}\right) + P(t_{pi} + t_{po})$$

$$= 2\,(9.096)\left(\frac{4.5}{1.6}\right) + (4.905 + 1)\left(5.4 - \frac{4.5}{1.6} + 2 + 3 + 0 - 0\right) + 6.4(1.2 + 1.2)$$

$$= 111.3996\ s$$

So, the value of the round trip time found using the exact value method equals 111.3996 s.

Using the Monte Carlo method, the number of passengers P equals the 80% car-rated capacity, hence P= 6.4.

Then the PDF, CDF graphs for the floor population percentage should be sketched. As the floor population is equal for all floors then $U = N\,(150) = 150\,(10) = 1500\ persons$

$U_k/U = \frac{150}{1500} = \frac{1}{10} = 0.1$, where k indicates floors from 1 to N.

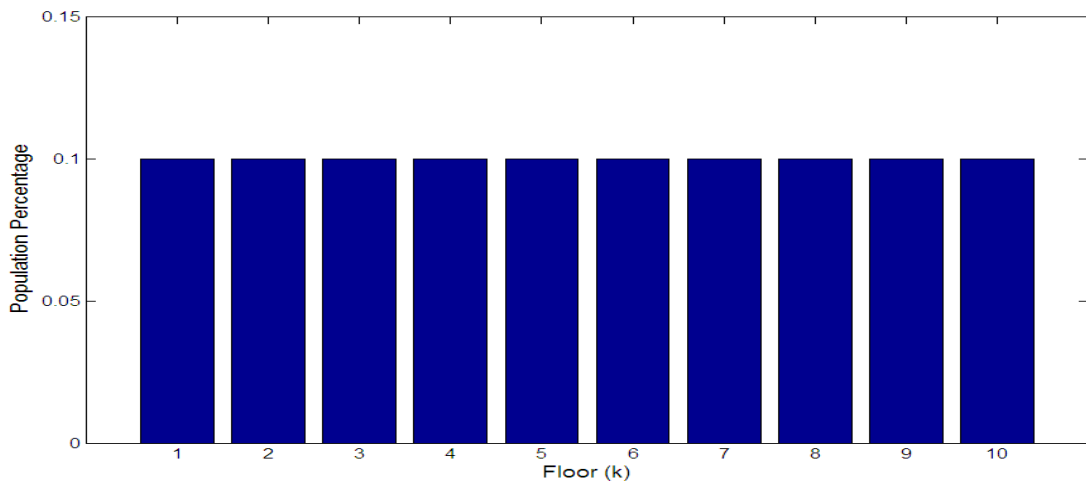The PDF and CDF graphs are sketched in figure 5.16 and figure 5.17 respectively.

Figure 5.16: The probability density function of the floors population percentage
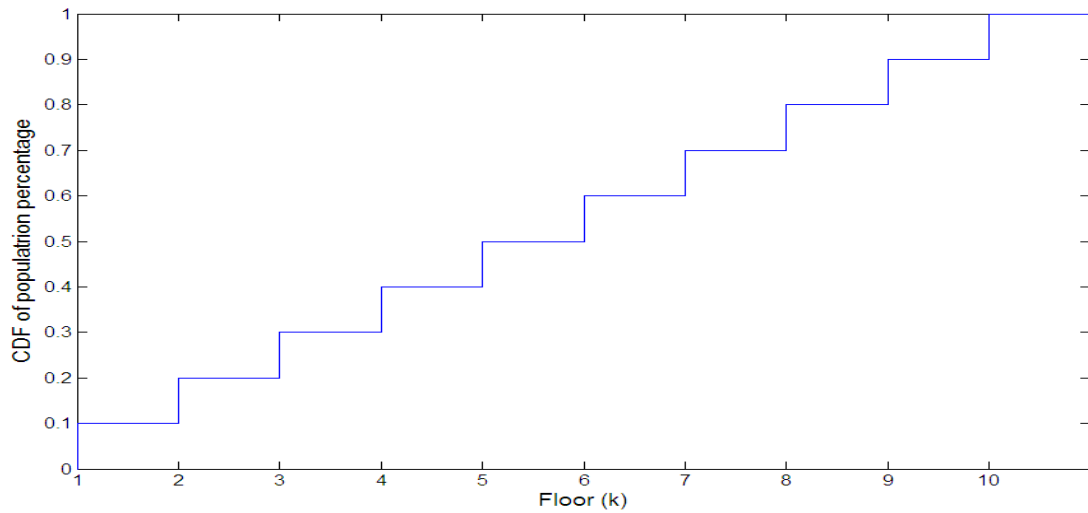
Figure 5.17: The Cumulative density function of the floors population percentage

The next step is to generate five random numbers between (0-1), the random numbers generated are: 0.093, 0.029, 0.455, 0.324, 0.113, 0.829, 0.845, 0.104, 0.466, 0.431

By scanning the CDF, the random generated floors based on the floor population percentage are: First floor, first floor, fifth floor, fourth floor, second floor, ninth floor, ninth floor, second floor, fifth floor and fifth floor. So there will be one stop on the first floor, one stop on the second floor, one stop on the fourth floor, one stop on the fifth floor and one stop on the ninth floor. Therefore, the number of stops equals five stops.

The kinematics time for this journey scenario is:

$$\tau_k = \tau_{01} + \tau_{12} + \tau_{24} + \tau_{45} + \tau_{59} + \tau_{90}$$

Where:

τ_{01} : is the transition time between the ground floor and the first floor.

τ_{12}: is the transition time between the first floor and the second floor.

τ_{24}: is the transition time between the second floor and the fourth floor.

τ_{45}: is the transition time between the fourth floor and the fifth floor.

τ_{59}: is the transition time between the fifth floor and the ninth floor.

τ_{90}: is the transition time between the ninth floor and the ground floor.

Since the top speed is attained in one floor journey the transition times are calculated as follows:

$$\tau_{01} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41 \, s$$

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$$\tau_{12} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41 \ s$$

$$\tau_{24} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5 \ (2)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 8.225 \ s$$

$$\tau_{45} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 5.41 \ s$$

$$\tau_{59} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5 \ (4)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 13.85 \ s$$

$$\tau_{90} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5 \ (10)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 30.725 \ s$$

The kinematics time equals to:

$$\tau_k = \ \tau_{01} + \tau_{12} + \tau_{24} + \tau_{45} + \tau_{59} + \tau_{90} = 5.41 + 5.41 + 8.225 + 5.41 + 13.85 + 30.725$$
$$= 69.03s$$

The constant time for this journey scenario is calculated as follows:

$$\tau_c = S \ (t_{do} + t_{dc}) + P \ (t_{pi} + t_{po}) = 5(2 + 3) + 6.4(1.2 + 1.2) = 40.36 \ s$$

The round trip time for one trial equal:

$$\tau = \tau_k + \tau_c = 69.03 + 40.36 = 109.39 \ s$$

Using the Monte Carlo software tool, the round trip time found for one trial equal: 102.01 s

It is important to recall that the round trip time value found for one trial differ from trial to another as the procedure is totally random and depends on the set of random numbers generated each time.

Using the Monte Carlo Markov chain method, first we shall calculate the transition matrix by calculating the probabilities of transitions using these two equations:

The probability of elevator transition between any two floors:

$$p \ (J_{ij}) = \left[\left(1 - \sum_{k=i+1}^{j-1} \left(\frac{U_k}{U} \right) \right)^p - \left(1 - \sum_{k=i}^{j-1} \left(\frac{U_k}{U} \right) \right)^p - \left(1 - \sum_{k=i+1}^{j} \left(\frac{U_k}{U} \right) \right)^p + \left(1 - \sum_{k=i}^{j} \left(\frac{U_k}{U} \right) \right)^p \right] \Big/ \left[1 - \left(1 - \frac{U_k}{U} \right)^p \right]$$

The probability of elevator transition between the main floor and any other floor:

$$P(J_{0j}) = \left(1 - \sum_{k=1}^{j-1} \left(\frac{U_k}{U} \right) \right)^P - \left(1 - \sum_{k=1}^{j} \left(\frac{U_k}{U} \right) \right)^P$$

The transition probabilities are calculated and filled in the transition matrix. The transition matrix is shown in table 5.4 below.

Table 5.4: the transition probability matrix for case study 2

	0	1	2	3	4	5	6	7	8	9	10
0	0	0.4905	0.2697	0.1378	0.0640	0.0262	0.0090	0.0024	4.1676e-04	3.3221e-05	3.9811e-07
1	8.1165e-07	0	0.4500	0.2691	0.1504	0.0770	0.0350	0.0135	0.0040	7.8194e-04	6.6919e-05
2	6.7731e-05	0	0	0.4500	0.2691	0.1504	0.0770	0.0350	0.0135	0.0040	7.8194e-04
3	8.4967e-04	0	0	0	0.4500	0.2691	0.1504	0.0770	0.0350	0.0135	0.0040
4	0.0049	0	0	0	0	0.4500	0.2691	0.1504	0.0770	0.0350	0.0135
5	0.0184	0	0	0	0	0	0.4500	0.2691	0.1504	0.0770	0.0350
6	0.0534	0	0	0	0	0	0	0.4500	0.2691	0.1504	0.0770
7	0.1304	0	0	0	0	0	0	0	0.4500	0.2691	0.1504
8	0.2808	0	0	0	0	0	0	0	0	0.4500	0.2691
9	0.5500	0	0	0	0	0	0	0	0	0	0.4500
10	1	0	0	0	0	0	0	0	0	0	0

After that, PDF and CDF graphs should be sketched for each row of the matrix, they are shown in figures below (figures 5.18- 5.39).
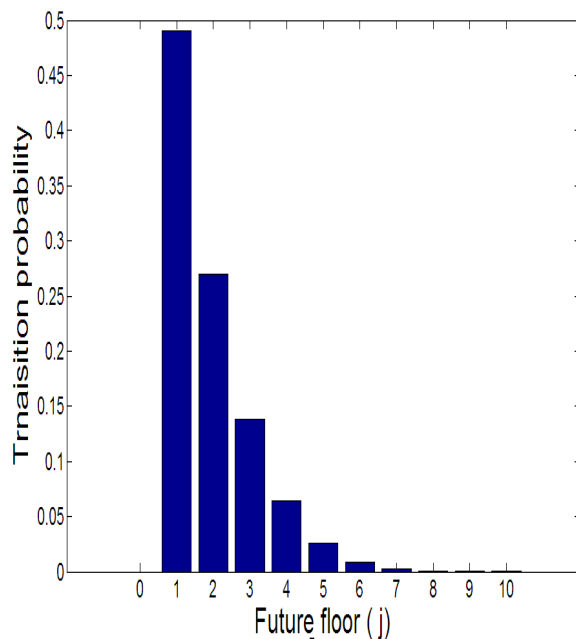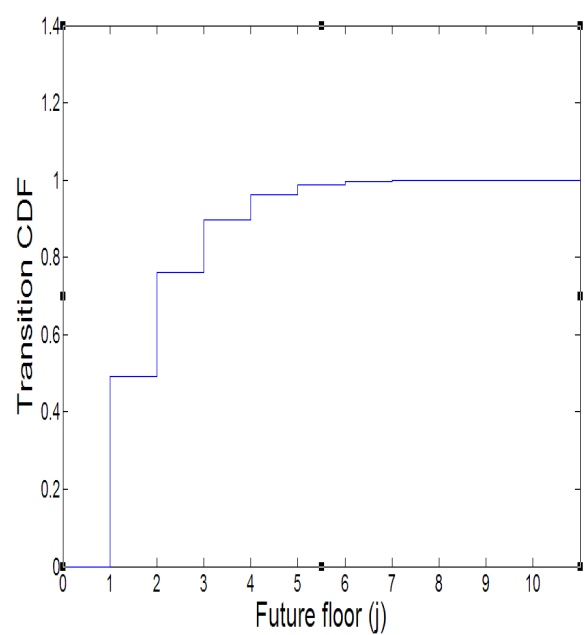

Figure 5.18: PDF of the ground floor

Figure 5.19: CDF of the ground floor

Figure 5.20: PDF of the first floor

Figure 5.21: CDF of the first floor

Figure 5.22: PDF of the second floor

Figure 5.23: CDF of the second floor

Figure 5.24: PDF of the third floor

Figure 5.25: CDF of the third floor

Figure 5.26: PDF of the fourth floor

Figure 5.27: CDF of the fourth floor

Figure 5.28: PDF of the fifth floor

Figure 5.29: CDF of the fifth floor

Figure 5.30: PDF of the sixth floor

Figure 5.31: CDF of the sixth floor

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Figure 5.32: PDF of the seventh floor

Figure 5.33: CDF of the seventh floor
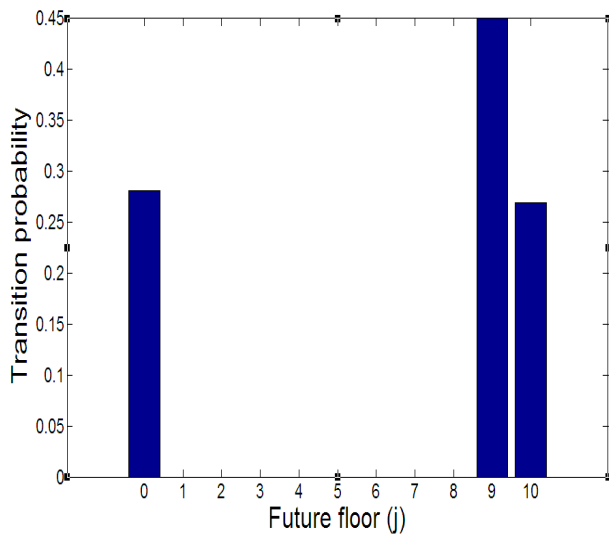
Figure 5.34: PDF of the eighth floor
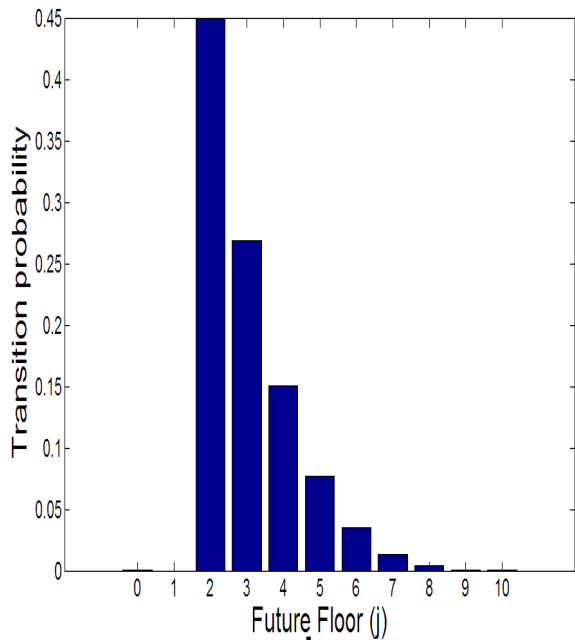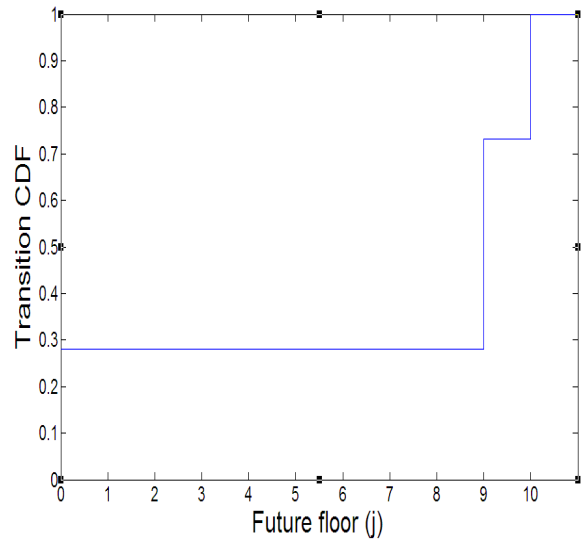

Figure 5.35: CDF of the eighth floor
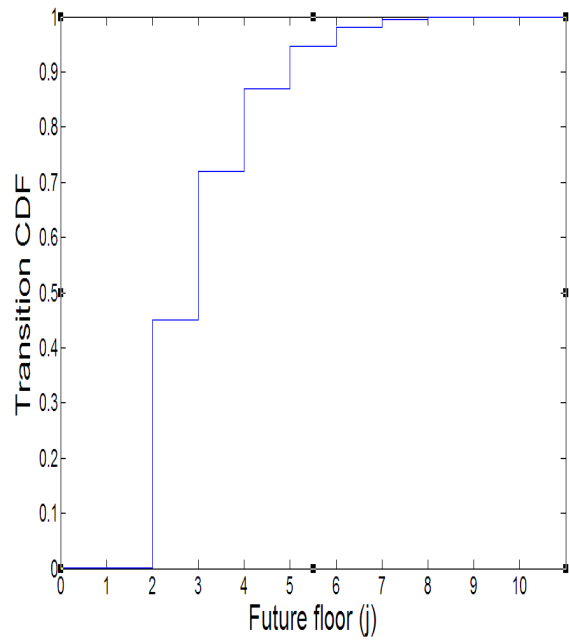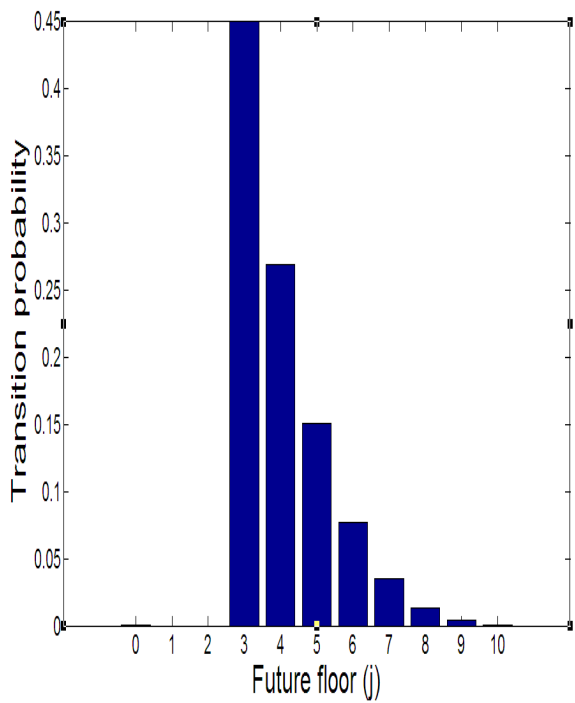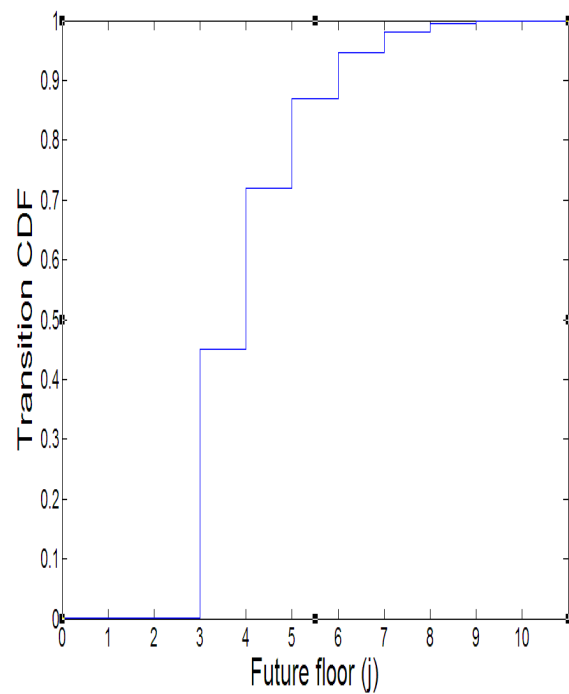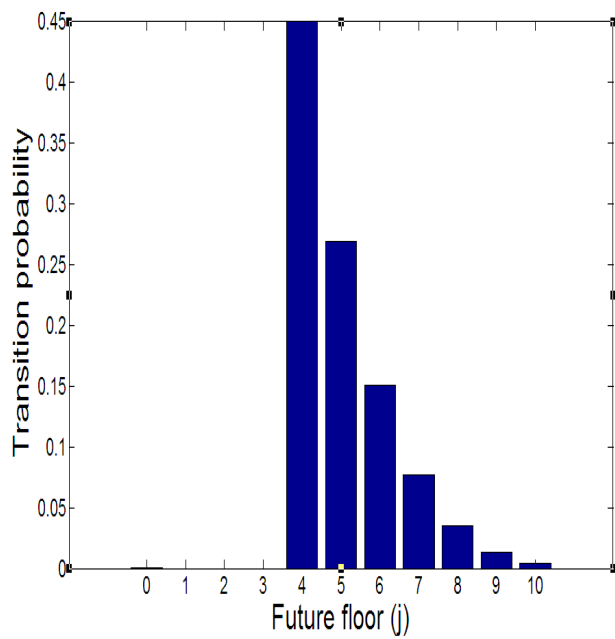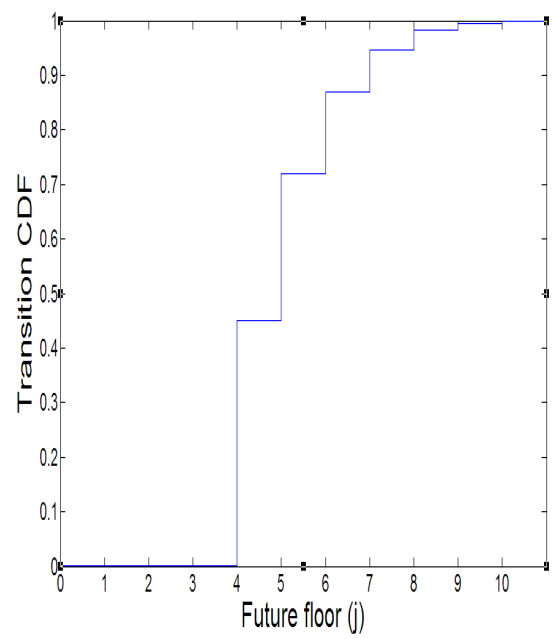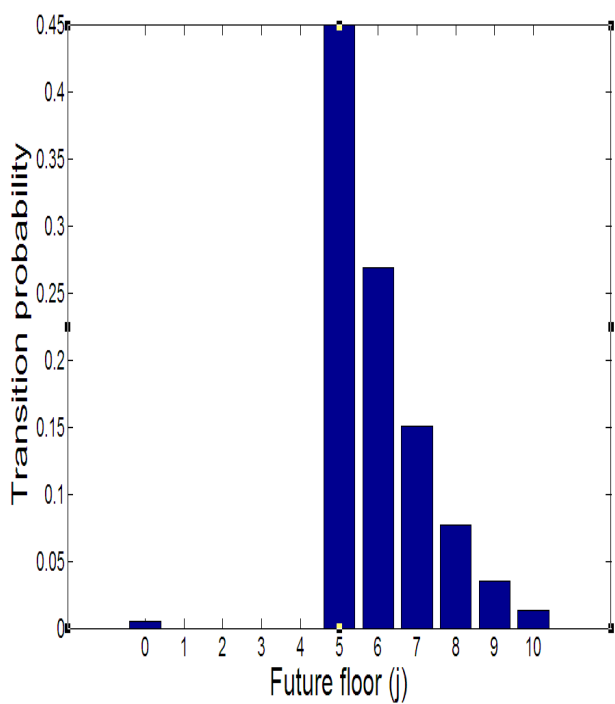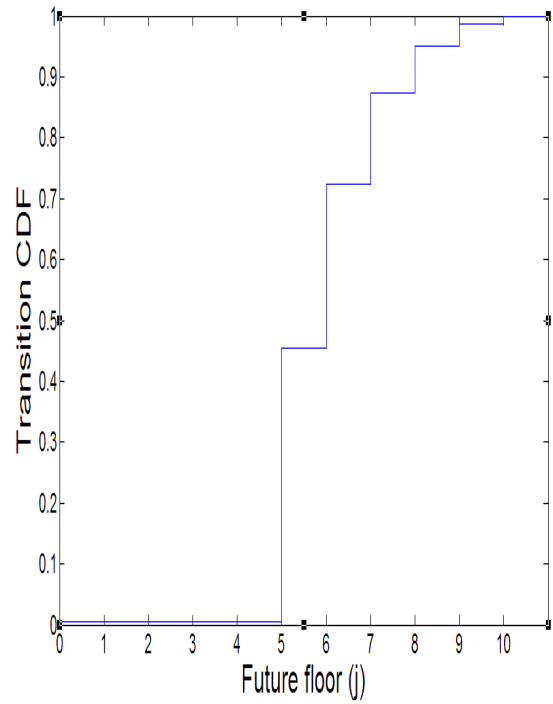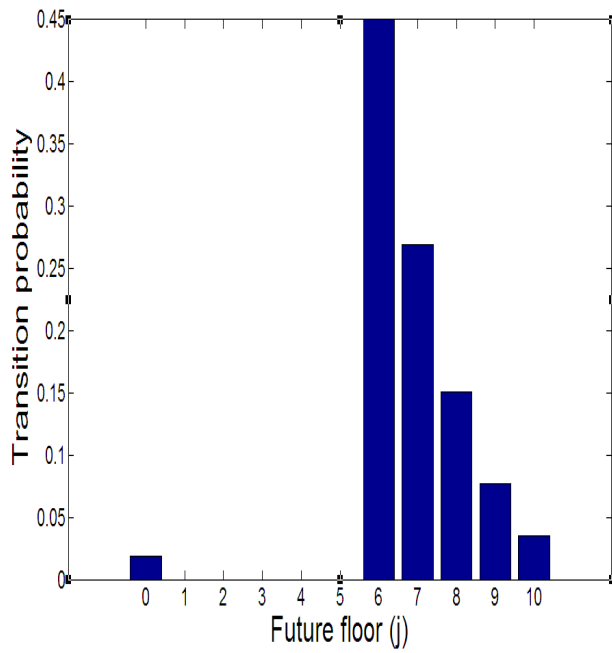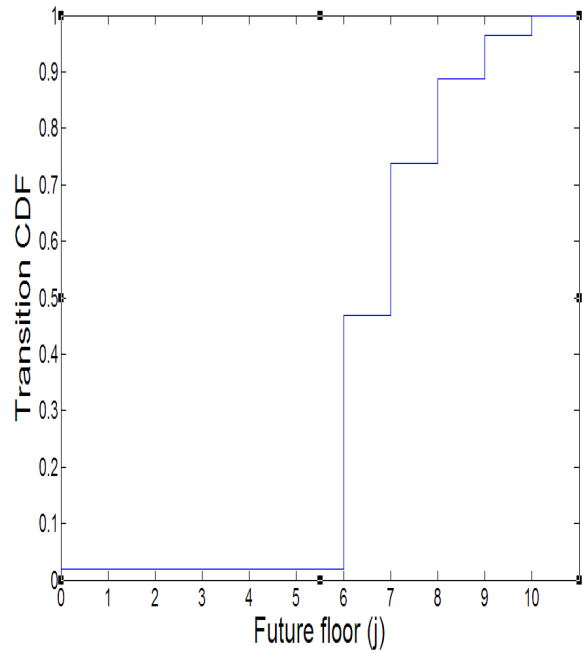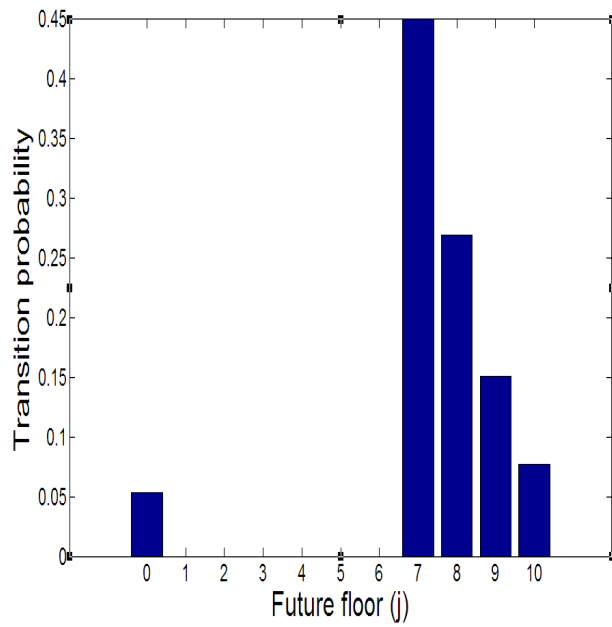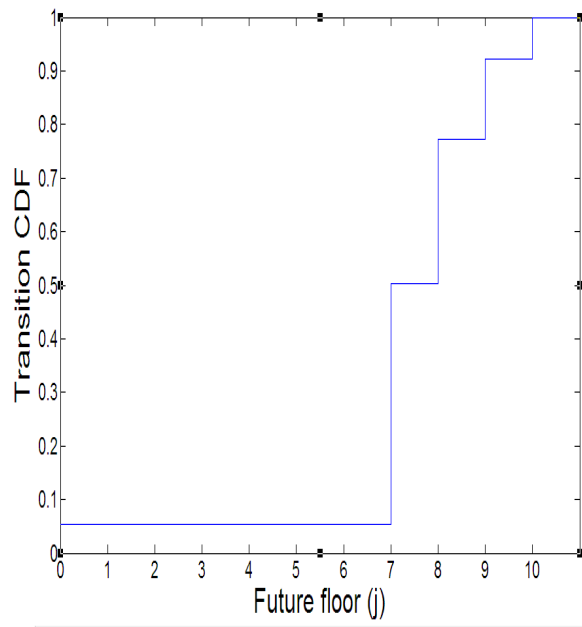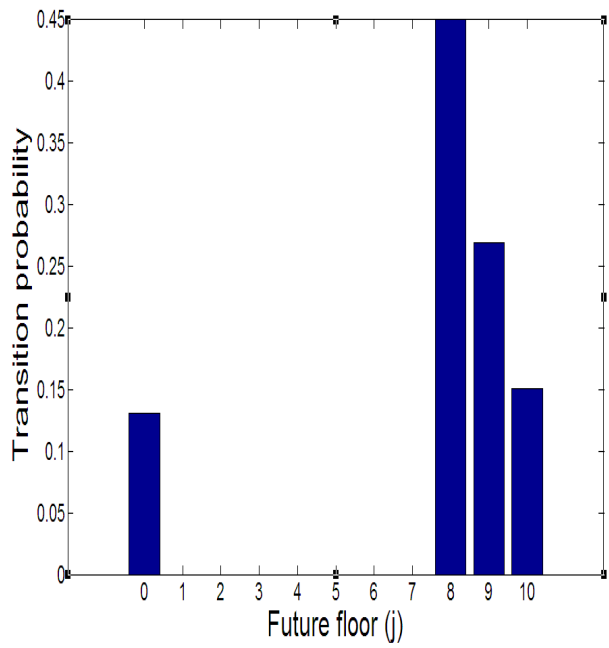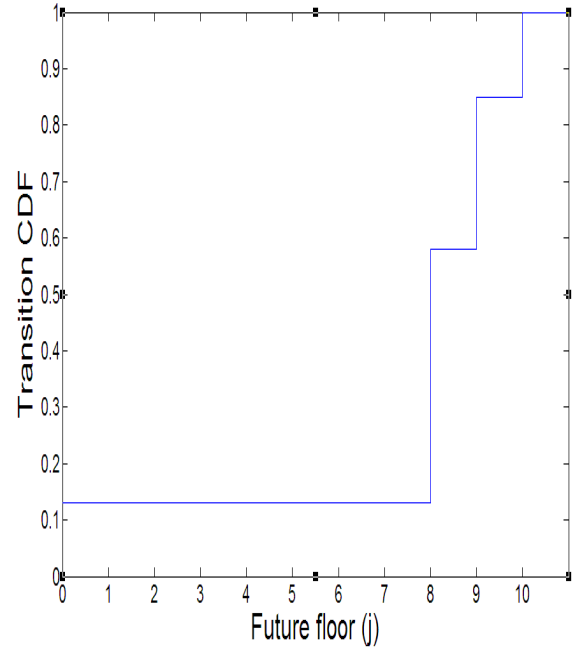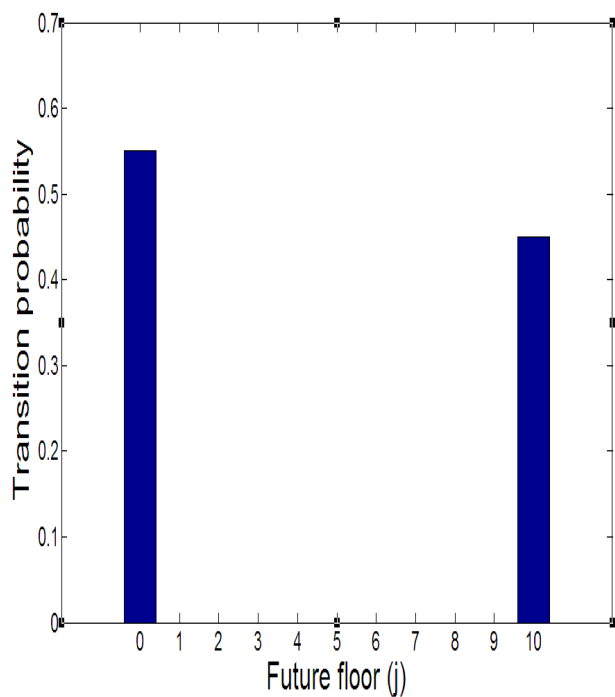
Figure 5.36: PDF of the ninth floor
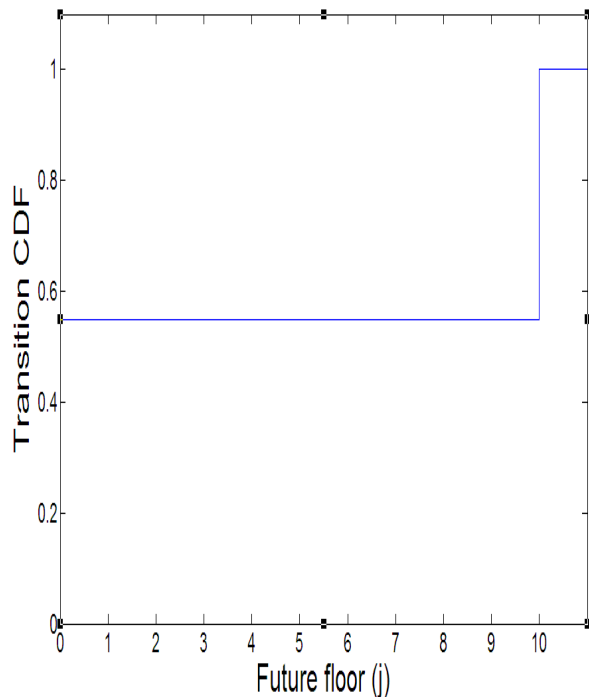
Figure 5.37: CDF of the ninth floor
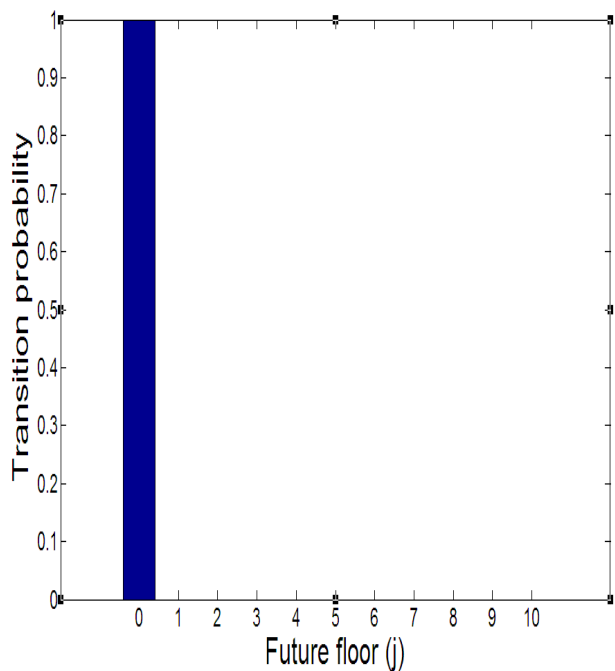
Figure 5.38: PDF of the tenth floor
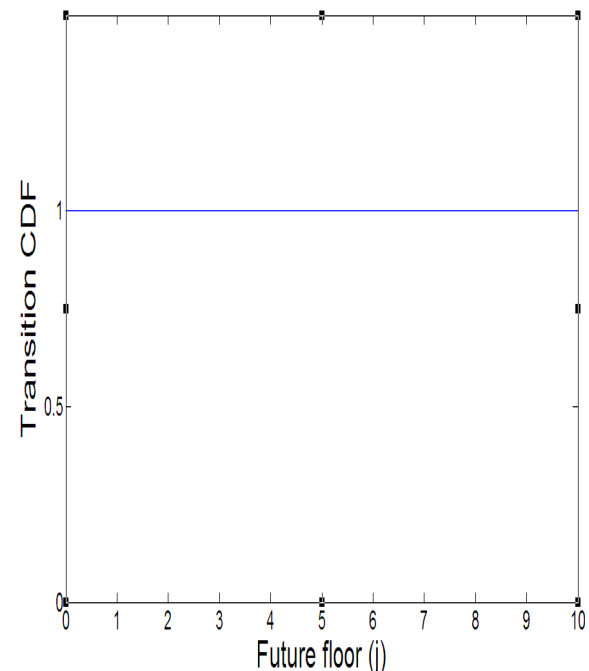
Figure 5.39: CDF of the tenth floor

Afterwards, the first random number is generated; the first random number is 0.684. By scanning the ground floor's CDF we have the first random destination which is the second floor. Taking another random number, we have 0.492, by scanning the second floor's CDF we have the next destination which is in this case is the fourth floor. Taking another random number we have 0.999 by scanning the fourth floor's CDF we get the next destination which in this case is the tenth floor. Taking another random number we have 0.057, by scanning the tenth floor CDF we have the next destination which is the ground floor.

Now, the first journey scenario is generated; where the elevator will stop first at the second floor then at the fourth floor then at the tenth floor before it turns back to the ground floor, hence the number of stops for this journey equals three. Number of passengers equals three too.

The kinematics time for this journey scenario is:

$$\tau_k = \tau_{02} + \tau_{24} + \tau_{4-10} + \tau_{10-0}$$

Where:

τ_{02} : is the transition time between the ground floor and the second floor.

τ_{24}: is the transition time between the second floor and the fourth floor.

τ_{4-10}: is the transition time between the fourth floor and the tenth floor.

τ_{10-0}: is the transition time between the tenth floor and the ground floor.

Since the top speed is attained in one floor journey the transition times are calculated as follows:

$$\tau_{02} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(2)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 8.225 \ s$$

$$\tau_{24} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5(2)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 8.225 \ s$$

$$\tau_{4-10} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5 \ (6)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 19.475 \ s$$

$$\tau_{10-0} = \frac{d}{v} + \frac{v}{a} + \frac{a}{j} = \frac{4.5 \ (10)}{1.6} + \frac{1.6}{1} + \frac{1}{1} = 30.725 \ s$$

The kinematics time equals to:

$$\tau_k = 8.225 + 8.225 + 19.475 + 30.725 = 66.65 \ s$$

The constant time for this journey scenario is calculated as follows:

$$\tau_c = S \ (t_{do} + \ t_{dc}) + P \ (t_{pi} + t_{po}) = 3(2 + 3) + 3(1.2 + 1.2) = 22.2 \ s$$

The round trip time for one trial equal:

$$\tau = \tau_k + \tau_c = 38.52 + 22.2 = 88.85 \, s$$

Using the Monte Carlo Markov chain software tool, the round trip time found for one trial equal: 109.61 s. It is important to recall that the round trip time value found for one trial differ from trial to another as the procedure is totally random and depends on the set of random numbers generated each time.

Using the simulation software, we will get the results shown in the table 5.5 below.

Table 5.5: the simulation software results for the values of round trip time using MCMC and MC methods for different number of trials for case study 2

Number of Trials	RTT using MCMC	RTT using Monte Carlo
100	108.3428 s	106.0108 s
1000	110.3737 s	107.9322 s
10000	111.4742 s	109.5695 s
100000	111.381 s	110.3372 s

After that, the percentage of error for Monte Carlo Markov Chain method is calculated as follows:

$$e\% = \left| \frac{exact \, value - MCMC \, value}{exact \, value} \right| \times 100\%$$

Likewise, the percentage of error for Monte Carlo method is calculated as follows:

$$e\% = \left| \frac{exact \, value - MC \, value}{exact \, value} \right| \times 100\%$$

The error percentage values are calculated and filled in table 5.6 below.

Table 5.6: percentage of error for Monte Carlo Markov chain method and Monte Carlo method for case study 2

Number of Trials	MCMC percentage of error	Monte Carlo percentage of error
100	2.744%	4.837%
1000	0.921%	3.113%
10000	0.067%	1.643%
100000	0.017%	0.954%

The percentage of error functions is sketched and shown in figure 5.40 below.
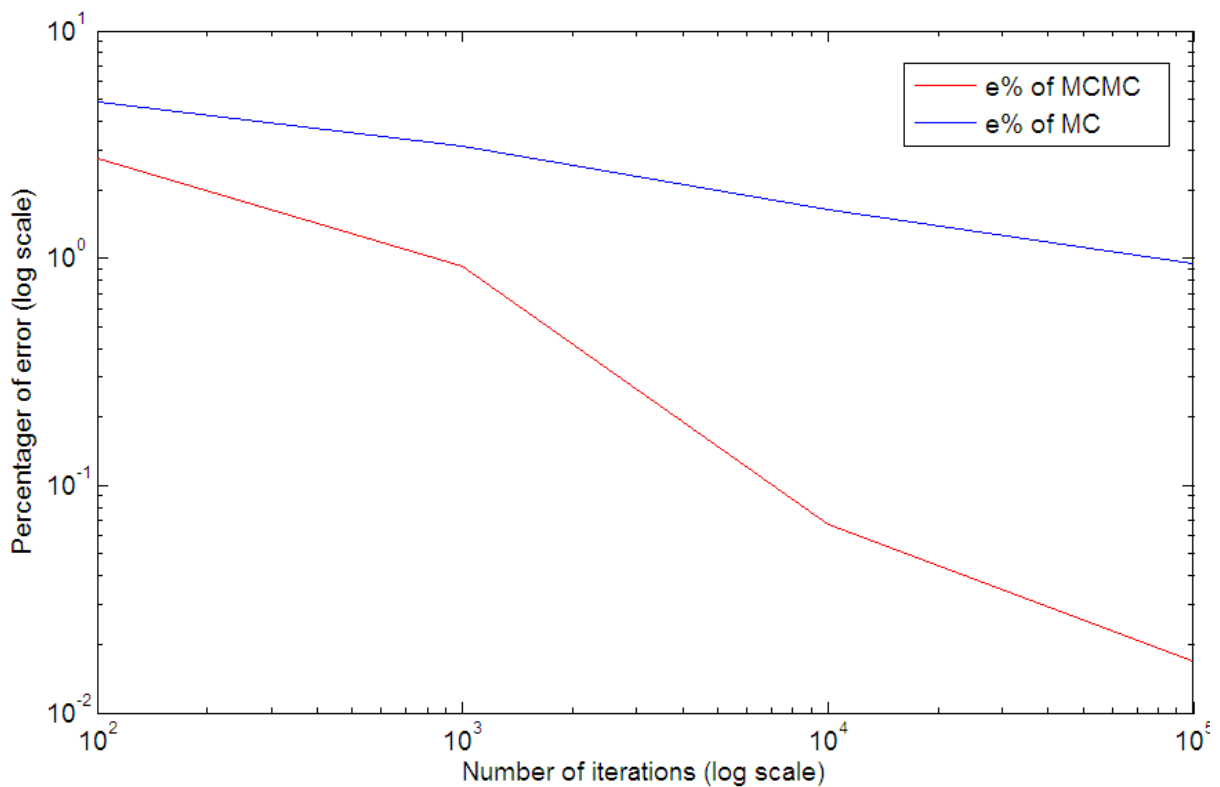
Figure 5.40 : Percentage of error versus number of trials in log scale

As clear from the results of the two case studies, Monte Carlo Markov Chain method is better than Monte Carlo method, as the Monte Carlo Markov chain method gave closer results to the exact value and a less percentage of error than the Monte Carlo method. It is also clear that the Monte Carlo Markov Chain methods approaches the exact value faster than the Monte Carlo method that needs larger number of trials to approach the exact value.

5.3   Conclusions and future work

The design of the elevator systems involves the selection of the number, capacity and speed of the elevators to achieve the required performance, and it relies on the calculation of the round trip time, as the round trip time gives a good measure for both the quantity and quality of service, as both the interval and the handling capacity calculations depends on the round trip time value. Therefore, it is important to derive accurate and easy methods to calculate the round trip time. The Monte Carlo Markov chain simulation method was introduced as a methodology to arrive at the value of the round trip time during incoming traffic conditions in the simplest case which assumes equal floor population, equal floor heights, one entrance floor and a top speed that is attained in a one floor journey. The advantages of the Monte Carlo Markov chain method is the simplicity of programming and accuracy.

Also, the Monte Carlo simulation method was introduced as a simulation method for calculating the round trip time. Monte Carlo Markov chain simulation method is better than Monte Carlo simulation method as the first one is faster and needs less number of trials to approach the exact value, and it also gives a less percentage or error.

Furthermore, If any or a combination of the general assumptions of; equal floor population, equal floor height, single entrance and the top speed attained in a one floor journey, was dropped, Monte Carlo Markov chain simulation method can be easily developed to cover all of these cases. However, the analytical method cannot deal with a combination of these special cases as the problem becomes very complicated. So, the development of the Monte Carlo Markov chain simulation method can give a good alternative for the analytical method for calculating the round trip time when any or all of the general four assumptions does not exist.

Likewise, in the future Monte Carlo Markov chain simulation method can be developed to a Markov Chain method which only depends on the Markov chains to arrive at the value of the round trip time depending only on the steady state probabilities.

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Appendix

Matlab GUI Code for the software tool

```matlab
N = str2num(get(handles.edit1,'String'));
u = str2num(get(handles.edit2,'String'));
Pa = str2num(get(handles.edit3,'String'));
tpi = str2num(get(handles.edit4,'String'));
tpo = str2num(get(handles.edit5,'String'));
tdo = str2num(get(handles.edit6,'String'));
tdc = str2num(get(handles.edit7,'String'));

U = u*N;
set (handles.edit8,'String',num2str(U));


%-----------------------------------------------------------------
%transition Matrix (A)  Construction
%-----------------------------------------------------------------

A = zeros(N+1);
A(N+1,1)= 1;

for i=1:N
    for j = 1:N+1

        if (i<j)

            if(i==1)
                c1=0;c2=0;
                for k=1:j-2
                    c1 = c1 + u/U;
                end
                for k=1:j-1
                    c2 = c2 + u/U;
                end

                 A(i,j) = ((1-c1)^Pa - (1-c2)^Pa);

            else
                k1=0;k2=0;k3=0;k4=0;
                for k=i:j-2
                    k1 = k1 + u/U;
                end
                for k=i-1:j-2
                    k2 = k2 + u/U;
```

```matlab
                end
                for k=i:j-1
                    k3 = k3 + u/U;
                end
                for k=i-1:j-1
                    k4 = k4 + u/U;
                end

                A(i,j) = ((1-k1)^Pa - (1-k2)^Pa -(1-k3)^Pa +(1-
k4)^Pa)/(1-(1-u/U)^Pa);
            end

        end
    end
end

for i = 2:N
    Row_sum = 0;
    for k = 2:N+1
        Row_sum = Row_sum + A(i,k);
    end

    A(i,1) = 1- Row_sum ;
end

A

set(handles.uitable1, 'Data', A);

%-------------------------------------------------------------
%steady state Probability (pi)
%-------------------------------------------------------------
IM = A-eye(N+1);
IM(:,N+1)=  ones(N+1,1);
R = zeros(1,N+1);
R(1,N+1) = 1;

X = R*inv(IM);
set(handles.uitable2, 'Data', X);

%-------------------------------------------------------------
%Analytical Solution
%-------------------------------------------------------------

df = str2num(get(handles.edit18,'String'));
v =str2num(get(handles.edit19,'String'));
a = str2num(get(handles.edit20,'String'));
```

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```matlab
je = str2num(get(handles.edit21,'String'));

tf = df/v + v/a + a/je ;
tv = df/v ;
ts = tf - tv + tdo +tdc;

S2 = N*(1-(1- (1/N))^Pa);
set (handles.edit25,'String',num2str(S2));


im2=0;
for k =1:N-1
   im2 = im2 + (k/N)^Pa;
end

H = N - im2;

T2 = 2*H*tv + ts*(S2+1) + Pa*(tpi +tpo);

set (handles.edit26,'String',num2str( T2 ));

%----------------------------------------------------------------
%simulation #1 : MARKOV simulation
%----------------------------------------------------------------
Nrpt = str2num(get(handles.edit31,'String'));

Tau_sum = 0;
for repeat = 1:Nrpt

s=0;
state1in = 0;
state2 = 1948;
Tk = 0;

while(state2 ~= 0)
    s = s+1;

state1 = state1in +1;
rndm = rand(1);
summ = 0;


for k =1:size(A,1)+1
    if summ >= rndm
        state2 = k-2 ;
        break;
    else
```

```matlab
        summ = summ + A(state1 , k);
    end
end

d = abs(state2 - state1in)*df;

direction = (state2 - state1in);

%---------------------------------------------------------------
%time from floor to floor
%---------------------------------------------------------------
        if d >= (((a^2)*v + (v^2)*je)/(a*je))
        t_a2b = d/v + v/a + a/je;

        elseif d < (2*a^3/je^2)
        t_a2b = (32*d/je)^(1/3);

        else
        t_a2b = a/je + ((4*d/a + (a/je)^2)^(0.5));

        end
%---------------------------------------------------------------
Tk = Tk + t_a2b;
state1in = state2;

end

t_constant = Pa*(tpi + tpo) + (s)*(tdo+tdc);
Tau = t_constant + Tk;
Tau_sum = Tau_sum + Tau;
end

avg_Tau = Tau_sum / Nrpt

set (handles.edit33,'String',num2str( avg_Tau ));


%----------------------------------------------------------------
%simulation #2 :Monte Carlo
%----------------------------------------------------------------
Tau2_sum = 0;
B = zeros(N+1);% # Passengers matrix
T = zeros(N+1);%# transitions matrix

for repeat = 1:Nrpt

Random = sort( rand(1,Pa) );
```

```
Randomx = 0;
Floor = [0, floor(N*Random +1) ];
transition = 0;

for k = 1:Pa

   if ( Floor(k+1) ~= Floor(k) );
        transition(length(transition)+1) = Floor(k+1);
     end
end

d= [diff(transition),max(transition)]*df;
    %------------------------------------------------------------
    %time from Floor to Floor
    %------------------------------------------------------------
Tk2 = 0;
for ln = 1: size(d,2)
    d_ln = d(ln);

        if d_ln >= ((a^2)*v + (v^2)*je)/(a*je)
        t_a2b = d_ln/v + v/a + a/je ;
        elseif d_ln < (2*a^3/je^2)
        t_a2b = (32*d_ln/je)^(1/3);
        else
        t_a2b = a/je + ((4*d_ln/a + (a/je)^2)^(0.5));
        end

        Tk2 = Tk2 + t_a2b;
end
%------------------------------------------------------------
Splus1 = size(transition,2);
t_constant2 = Pa*(tpi + tpo) + (Splus1)*(tdo+tdc);
Tau2 = t_constant2 + Tk2;
Tau2_sum = Tau2_sum + Tau2;


%------------------------------------------------------------
%Monte Carlo simulated Matrix
%------------------------------------------------------------

%B = zeros(N+1); initially
transition(length(transition)+1) = 0;
%B is the number of passengers moved from floor =(row-1) to
floor =(column-1)
%T is the number of transitions from floor =(row-1) to floor
=(column-1)
for j=1:length(transition)-1
```

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```matlab
        B(transition(j)+1,transition(j+1)+1) =
        B(transition(j)+1,transition(j+1)+1) + sum(Floor ==
        transition(j+1));
        T(transition(j)+1,transition(j+1)+1) =
        T(transition(j)+1,transition(j+1)+1) + 1;
    end
    %B
    %T
    %-----------------------------------------------------------
    %-----------------------------------------------------------
end
B
T=T/Nrpt
set(handles.uitable4, 'Data', T);
%-----------------------------------------------------------
%-----------------------------------------------------------
avg_Tau2 = Tau2_sum / Nrpt

set (handles.edit34,'String',num2str( avg_Tau2 ));

set (handles.text44,'String','ATH');
%-----------------------------------------------------------
%-----------------------------------------------------------

%Markov from Monte Carlo

%------------------------------------------------------------
%transition Matrix (P)  Construction
%------------------------------------------------------------

for i=1:N+1
    Added(i)= sum(T(i,:));
    %T(i,1)=T(i,1) + 1- sum(T(i,:));
end
Production = 1/min(Added);
T(2:N+1,:)=T(2:N+1,:)*Production;
P = T
%set(handles.uitable7, 'Data', A);

%------------------------------------------------------------
%simulation #1 : MARKOV simulation
%------------------------------------------------------------
Nrpt = str2num(get(handles.edit31,'String'));

Tau_sum = 0;
for repeat = 1:Nrpt
```

```matlab
s=0;
state1in = 0;
state2 = 1948;
Tk = 0;

while(state2 ~= 0)
    s = s+1;
state1 = state1in +1;
rndm = rand(1);
summ = 0;
for k =1:size(P,1)+1
    if summ >= rndm
        state2 = k-2;
        break;
    else
        summ = summ + P(state1 , k);
    end
end
d = abs(state2 - state1in)*df;
direction = (state2 - state1in);
%------------------------------------------------------------
%time from floor to floor
%------------------------------------------------------------
        if d >= (((a^2)*v + (v^2)*je)/(a*je))
        t_a2b = d/v + v/a + a/je;
        elseif d < (2*a^3/je^2)
        t_a2b = (32*d/je)^(1/3);
        else
        t_a2b = a/je + ((4*d/a + (a/je)^2)^(0.5));
        end
%------------------------------------------------------------
Tk = Tk + t_a2b;
state1in = state2;
end
t_constant = Pa*(tpi + tpo) + (s)*(tdo+tdc);
Tau = t_constant + Tk;
Tau_sum = Tau_sum + Tau;
end
avg_Tau = Tau_sum / Nrpt
```